Black Holes in non-perturbative Quantum Gravity

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past works with Alexei Starobinsky and works in progress
with Yi-Fu Cai, A.Naskar, L.Rachwal, A.Tokareva and my students

Breakdown of the problem

UV complete gravity – already a challenge for more than a century

- Many attempts, no complete satisfaction yet

Infinite derivatives

- General considerations and, for example, Asymptotic Safety suggest infinite derivative Lagrangians

Strings

- Strings and especially string field theory strongly suggest non-local interactions in the form of infinite-derivative form factors

Aref'eva, Barvinskiy, Biswas, Dragovich, Koivisto, Krasnikov, Kuz'min, Mazumdar, Modesto, Percacci, Platania, Saueressig, Sen, Siegel, Shapiro, Tomboulis, Weinberg, Witten, Zwiebach, . . .

Some old references

- **Classic one:**

  M. Ostrogradski, Mem. Ac. St. Petersburg, VI 4, 385–517 (1850)

- **Mathematical:**

- H.T. Davis, Ann. of Math. 2, no. 4, 686–714 (1931)

- H.T. Davis, The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order (Indiana, the Principia Press, 1936)

- R.D. Carmichael, Bull. Amer. Math. Soc. 42, 193–218 (1936)

- L. Carleson, Math. Scand. 1, 31–38 (1953)

- **Physical:**

- A. Pais and G.E. Uhlenbeck, Phys. Rev. 79, 145–165 (1950)

"Convergence" (renormalizability), "definite norm" (unitarity) and causality – cannot be achieved simultaneously. Fine, but what if violation of microcausality is hidden under the uncertainty scale? de Rham, Tokareva, Tolley, . . .

Action to study [1602.08475, 1606.01250, 1711.08864]

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda$$
$$+\frac{\lambda}{2}\Big(R\mathcal{F}_R(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\Big)\Bigg)$$

Here $\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$ with all f_{X_n} constants and we often use $\mathcal{F} \equiv \mathcal{F}_R$

We assume that $\Box$ enters form-factors in a combination $\Box/\mathcal{M}_s^2$ where the mass parameter is the non-locality scale. We put $\mathcal{M}_s = 1$ for a while.

This is the most general action (still redundant, $\mathcal{F}_2$ can be zero in $D = 4$ or a constant in $D > 4$) to study linear perturbations around MSS.

We name it Analytic Infinite Derivative (AID) gravity.

Covariant spin-2 propagator on MSS:

$$S_2 = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda\mathcal{F}_W\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

The Stelle's case corresponds to $\mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda\cdot 1\cdot\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$
$$= \frac{2\lambda}{M_P^2}(\bar{\Box} - m^2)$$

This is an obvious second pole which will be a ghost.

Physical propagators around Minkowski, AID form-factors:

$$\mathcal{O}_s = \frac{(6\lambda\Box\mathcal{F}(\Box) - M_P^2)(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1)}{6\lambda(\mathcal{F}(\Box) + \frac{1}{3}\mathcal{F}_W(\Box))}$$
$$= (\Box - \mu^2)e^{2\sigma(\Box)}$$

$$\mathcal{O}_t = \Box(2\lambda\Box\mathcal{F}_W(\Box)/M_P^2 + 1) = \Box e^{2\omega(\Box)}$$

Then, avoiding all odds $\omega = \sigma + \mathrm{const}$:

$$\mathcal{F}_W(\Box) = M_P^2 \frac{e^{2\omega(\Box)} - 1}{2\lambda\Box}$$

$$\mathcal{F}(\Box) = \frac{M_P^2}{6\lambda\Box}\left[\left(\frac{\Box}{\mu^2} - 1\right)e^{2\omega(\Box)} + 1\right]$$

What else can AID quadratic action serve for?

- If we just start with the above proposed quadratic in curvature action it can accommodate many interesting solutions without requiring any other more general gravity model.
- For example, any conformally flat metric which satisfies $\Box R = r_1 R$ with constant r_1 is a solution.
- In particular, Starobinsky inflation is an exact solution here.
- Solution representing a ghost-free bouncing scenarios also were found.

Recap on infinite derivatives

- **Graviton propagator in general is modified to**

$$\Pi = \frac{e^{2\omega(k^2)}}{k^2}$$

and $\omega(k^2)$ must be an entire function.

- **We thus must have an infinite number of derivatives**
- **Wick rotation is a problem but it got a resolution thanks to Pius, Sen, and also [arxiv:2103.01945]**
- **Theory is renormalizable and unitary.**
- **Full propagator yet to be computed.**

Action again

$$S = \int d^4x \sqrt{-g} \Bigg(\frac{M_P^2 R}{2}$$
$$+ \frac{\lambda}{2} \left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)R^{\mu\nu\lambda\sigma} \right) \Bigg)$$

A la Gauss-Bonnet combination $\mathcal{F}_1 = \mathcal{F}_4 = \mathcal{F},\ \mathcal{F}_2 = -4\mathcal{F}$ does not contribute to a propagator.
And we do not understand why!?

If $\mathcal{F}_4 \neq 0$ than a Schwarzschild BH is not a solution. Even if $\mathcal{F}_4 = 0$ we claim it is not!

WHY?

Equations and BH-s

Typical terms in EOM-s (trace equation):

$$M_P^2 R - 6\lambda\Box\mathcal{F}_1(\Box)R$$
$$-2\lambda\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Box^l R \Box^{n-l} R + \{8 \text{ terms}\} = -T$$

and T is the trace of the stress tensor.

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

Schwarzschild metric

$$A(r) = 1 - \frac{2GM}{r}$$

Schwarzschild BH: to be or not to be?

Without the Riemann tensor in equations naively yes.
Then why not?

Regularization

$$A(r) = 1 - \frac{2GM}{r} \to A(r) = 1 - \frac{2GM}{r}\tilde{A}(r,\alpha), \;\; \tilde{A} = e^{-\alpha/r^p}$$

such that $\tilde{A}(\infty) = 1, \tilde{A}(r)/r \to 0$ at zero and $\tilde{A}(r,0) = 1$.

We plug a regularized function in EOM-s and compute T – the stress tensor trace.

And then we compute $\int d^3x\sqrt{-g}T = E$ which is the total energy of the object. In static case it is its mass.

To simplify computations we actually compute

$$\lim_{\Delta t\to\infty}\frac{1}{2\Delta t}\int_{-\Delta t}^{\Delta t} dtd^3x\sqrt{-g}T$$

Schwarzschild BH in higher-derivative theories

Computing the total energy we yield (W is Weyl tensor)

$$-E = M - 4\pi\lambda \int_0^\infty r^2 dr \left(R\Box\mathcal{F}_1'(\Box)R + R_{\mu\nu}\Box\mathcal{F}_2'(\Box)R^{\mu\nu} + W_{\mu\nu\lambda\sigma}\Box\mathcal{F}_4'(\Box)W^{\mu\nu\lambda\sigma}\right)$$

If $\mathcal{F}(\Box) = \mathrm{const}$ (Stelle gravity) then it does not contribute.
Schematically

$$-E = M - 4\pi\lambda(E_0 + E_1 + E_2 + \dots)$$

Here E_n corresponds to $\Box^n$ and for $p = 1$

$$E_0 \sim 1/\alpha^3, \quad E_1 \sim 1/\alpha^6 + 1/\alpha^5, \quad \dots$$

E_0 comes from $\mathcal{F}(\Box) \sim \log(\Box)$

Convergence analysis

We can deduce that for the total energy

$$-E = M - 4\pi\lambda M^2(2\alpha)^{-\frac{3}{p}}\left(\sum_{n=0}^{\infty}(-1)^n\hat{f}_n\beta_n(p)(2\alpha)^{-\frac{2n}{p}} + \{2\text{ terms}\}\right) + O(M^3)$$

Here $\hat{f}_n = nf_n$ and $\hat{f}_0$ comes from a log.
Recall that $\mathcal{F}(\Box) = \sum_{n\geq 0} f_n\Box^n$.
The above series *can* converge if it is alternating with rapidly falling coefficients. Example

$$\sum_{k\geq 0}\frac{(-1)^k}{k!\alpha^k} = e^{-1/\alpha} \xrightarrow{\alpha\to 0} 0$$

Convergence analysis continued

By direct computations we can see that β_n grow rapidly.
The series for E will converge for any p if

$$\lim_{n\to\infty} \frac{|\hat{f}_n|}{e^{qn\log n}} = 0, \text{ for any } q > 0$$

For an entire function its maximal grows rate for large z is given by e^{sz^ρ}. ρ is the order and s is the type.
Computing β_n we find an acceptable order of $\mathcal{F}(\Box)$ is $\rho < 3/2$

However, from the perspective of QFT for renormalizbility and unitarity we need that $\mathcal{F}(\Box)$ grows at most polynomially along the positive real axis and this implies that the order of $\mathcal{F}(\Box)$ is infinite.

BH results briefly and what about micro-BH?

- Regularization approach is motivated by a collapse consideration. You must be able to form a BH starting with a regular matter distribution.
- Regularization of a Schwarzschild BH can be removed only in 2 and 4 derivative gravity. Any higher (finite) derivative gravity cannot have this solution.
- Infinite derivative case results in infinitely many terms like $1/\alpha^n$ and in principle a summation over n may have a good $\alpha \to 0$ limit.
  BUT this is NOT compatible with a viable propagator for a UV complete unitary gravity.
- We thus must accept that a UV complete gravity not only resolves the BH singularity but also limits the micro-BH mass from below to $\mathcal{M}_s$ which obeys $M_{inf} \ll \mathcal{M}_s < M_P$

Conclusions

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **This gravity model features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **We argue that these theories disregard singular BH solutions on the example of Schwarzschild BH.**

Future directions

- Other BH solutions (charged, extremal, rotating) should be analyzed.
- BH regularity as a given feature implies that QNM may be modified.
- QNM will not test the interior of a BH as such, but higher derivatives in the action will result in new QNM shapes which is a very interesting way to support the idea that a UV complete gravity resolves BH singularities.
- Mass inflation problem should be addressed

Thank you for listening!

Can it be a Non-local scalar field [arxiv:2103.01945]

Consider Analytic Infinite Derivative (AID) scalar field action:

$$L = \frac{1}{2}\phi(\Box - m^2)f^{-1}(\Box)\phi - V(\phi)$$

We demand the form-factor to be an exponent of an entire function $\sigma(z)$

$$f(z) = \exp(2\sigma(z))$$

This is required to have no extra poles in the perturbative vacuum.
We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

Non-local scalar field, continued

Several arguments to consider the above action:

- **It naturally appears in SFT and in p-adic strings**
- **It was proven to be unavoidable in order to build unitary and renormalizable diffeomorphism invariant gravity**
- **This construction can make any arbitrary potential renormalizable**
- **Surely, some other benefits**

Namely, we can adjust the fall rate of the propagator for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

New excitations – Half of them are ghosts!

Linearization around a background solution ϕ_0:

$$L = \frac{1}{2}\psi\left[(\Box - m^2)f^{-1}(\Box) - V''(\phi_0)\right]\psi$$

Let's assume $V''(\phi_0) = v \approx \text{const} \neq 0$.

- **In general there is an infinite number of new excitations with perhaps complex conjugate masses squared**
- **The kinetic operator is again an entire function and obeys the Weierstrass decomposition**

$$(\Box - m^2)f^{-1}(\Box) - v^2 \sim \prod_i (\Box - \mu_i^2)e^{\sigma_v(\Box)}$$

- **Each μ_i corresponds to a mass of a distinct excitation.**