Constructing Ricci-Flat Mirror Hypersurfaces Within *Spaces of General Type*

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w/Per Berglund

🙏: *Mikiya Masuda*

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Ricci-Flat Mirror Hypersurfaces

Playbill

The Story so Far…

Fusing Fugue

Meromorphic March

Mirror Minuet

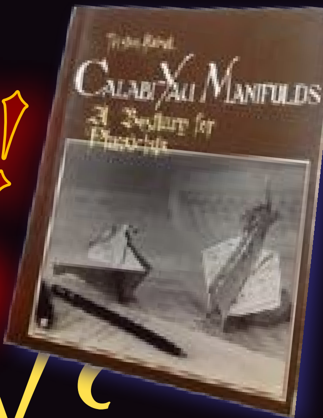
New? Toric Spaces

* "It doesn't matter what it's called,
...if it has substance."
— S.-T. Yau

worldsheet SCFT ground-states compactification

What-Where-Why?

32 nd! B-day



How?

$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \iff R_{\mu\nu} = 0 = T_{\mu\nu}$: vacua

nice "ambient space" $= \prod_i \mathbb{P}^{n_i},\ \mathbb{P}^{n_i}_{\vec{w}}$, toric…

- Complete Intersection: $X = \big(\cap_i \{f_i(z) = 0\}\big) \subset A$
- Constrained subspace: $\mathfrak{X}_i = \{f_i(z) = 0\} \subset A$
- Functions: $\mathcal{O}_X(d) \ni \phi_X(z) \simeq \big[\, \phi_A(z) \pmod{f_i(z)} \,\big]$ — generated by the defining system
  - *Calculus*: $T^*_X(d) \ni \mathrm{d}z_X \simeq \big[\, \mathrm{d}z_A \pmod{\mathrm{d}f_i(z)} \,\big]$—*"adjunction theorem"*
  - *Transversality*: $\{\wedge_i\, \mathrm{d}f_i \neq 0\} \cap \{f_i = 0\} \not\subset A$
- Anomaly-free: $\mathrm{d}^{\dim X} z_X \overset{!}{=} \mathcal{O}_X(0) \iff \deg[\mathrm{d}^{\dim A} z_A] \overset{!}{=} \deg[\mathrm{d}^K f_i]$
- Massless fields: $H^{p,q}(X) = H^q(X, \wedge^p T^*_X)$, also $H^q(X, \mathrm{End}\, T_X)$
  - "Gauge" (for "gauge" (for…)) equivalence classes
  - Bott-Borel-Weil: $\mathbb{P}^n = \frac{U(n+1)}{U(n)\times U(1)} \Rightarrow \phi^{a_1\cdots}_{b_1\cdots}(z_X) \sim U(n+1)$ **tensor expressions**

& Eastwood

worldsheet SCFT ground-states compactification

What-Where-Why?

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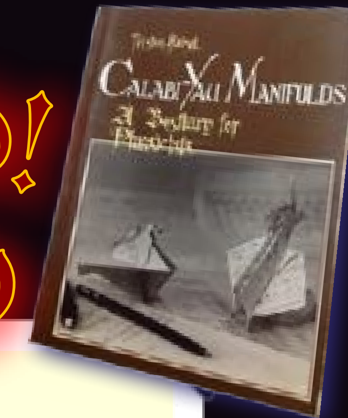
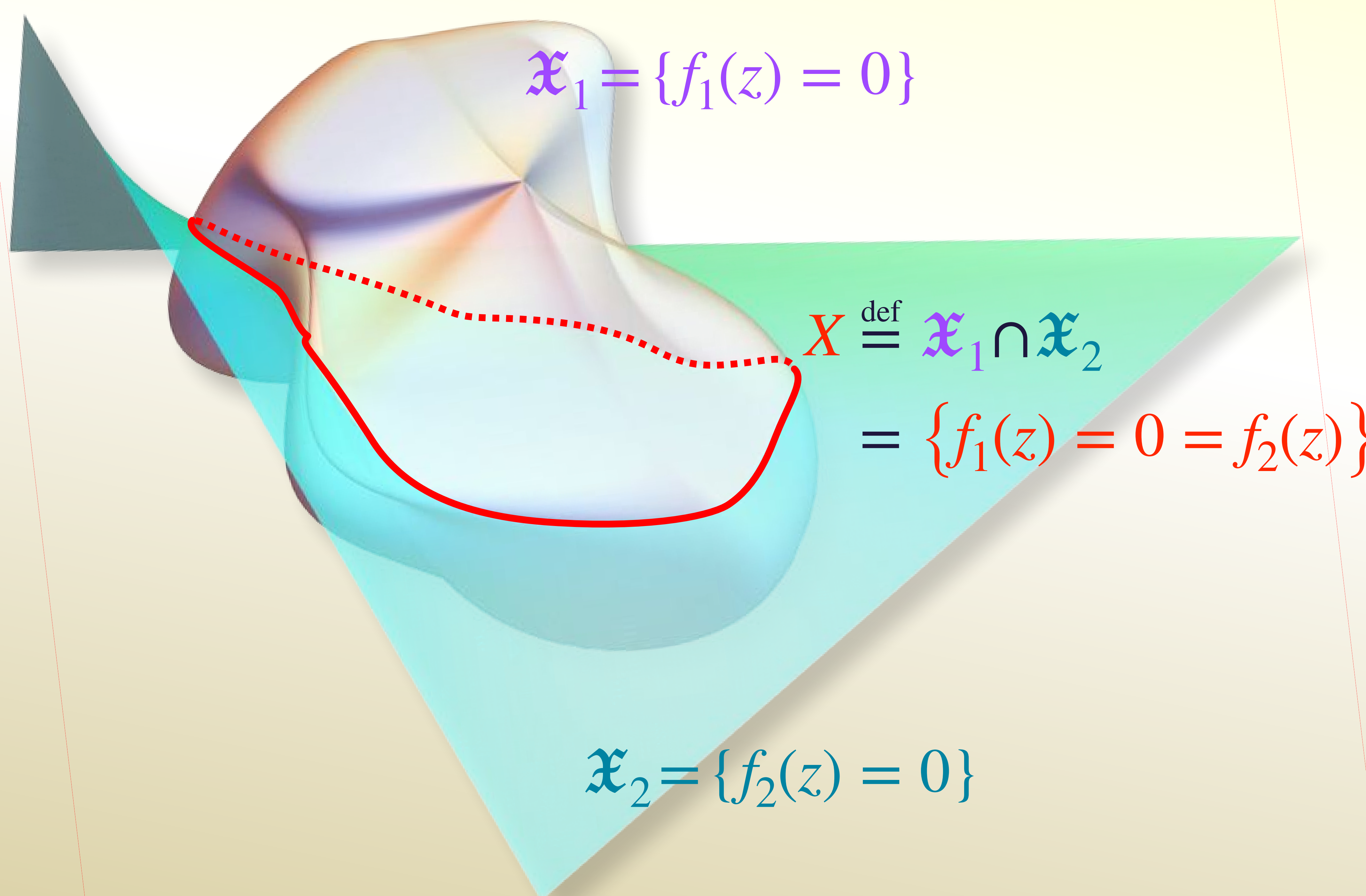


A picture is worth a thousand equations



$R_{\mu\nu}$ … ace" … c…

- Co…
- C…
- Fu…
- Ar…
- M…

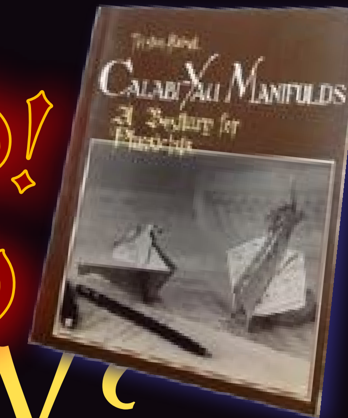
…ne …n

…ions

worldsheet SCFT ground-states compactification

What-Where-Why?

32 nd! B-day



How?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \iff R_{\mu\nu} = 0 = T_{\mu\nu} \ : \text{vacua}$$

→ ***nice*** "ambient space" $= \prod_i \mathbb{P}^{n_i},\ \mathbb{P}^{n_i}_{\vec{w}}$, toric…

Tian-Yau: $\{\text{Fano}\}_c \smallsetminus \{\text{CY}\}_c = \{\text{CY}\}_{nc}$

Also: $\{\mathscr{K}^*_{X_c}\} = \{\text{CY}\}_{nc}$

- Complete Intersection: $X = \big(\cap_i \{ f_i(z) = 0 \} \big) \subset A$
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& Eastwood

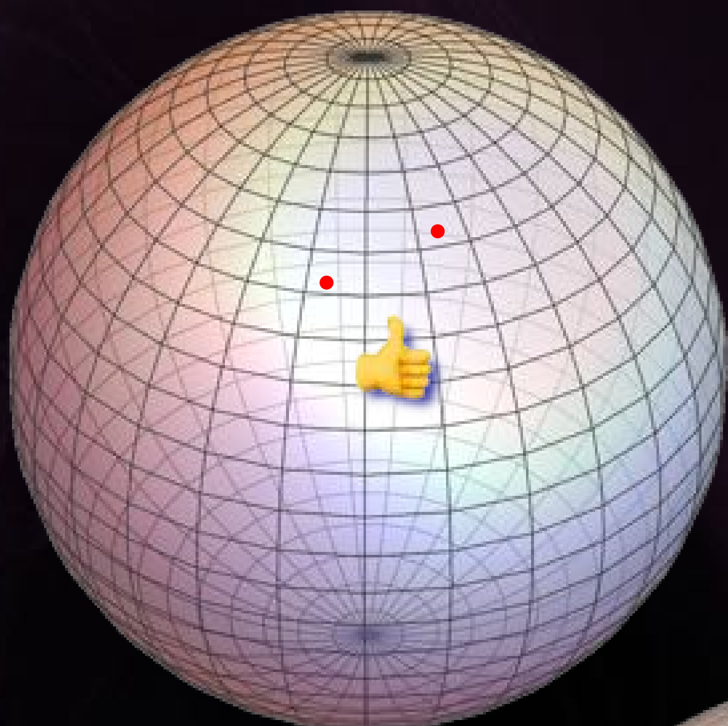
\+ `Macaulay2, SAGE, Magma,...` *(new tricks/old dogs…)*

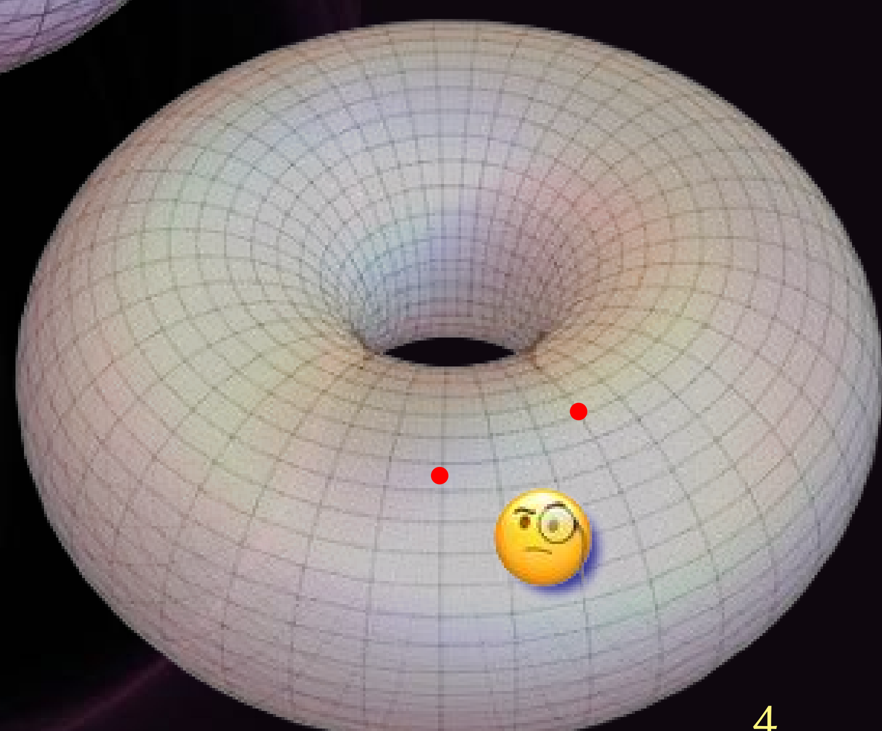
→ *sequel: "old dogs strike back" & ML/NN-mertics*

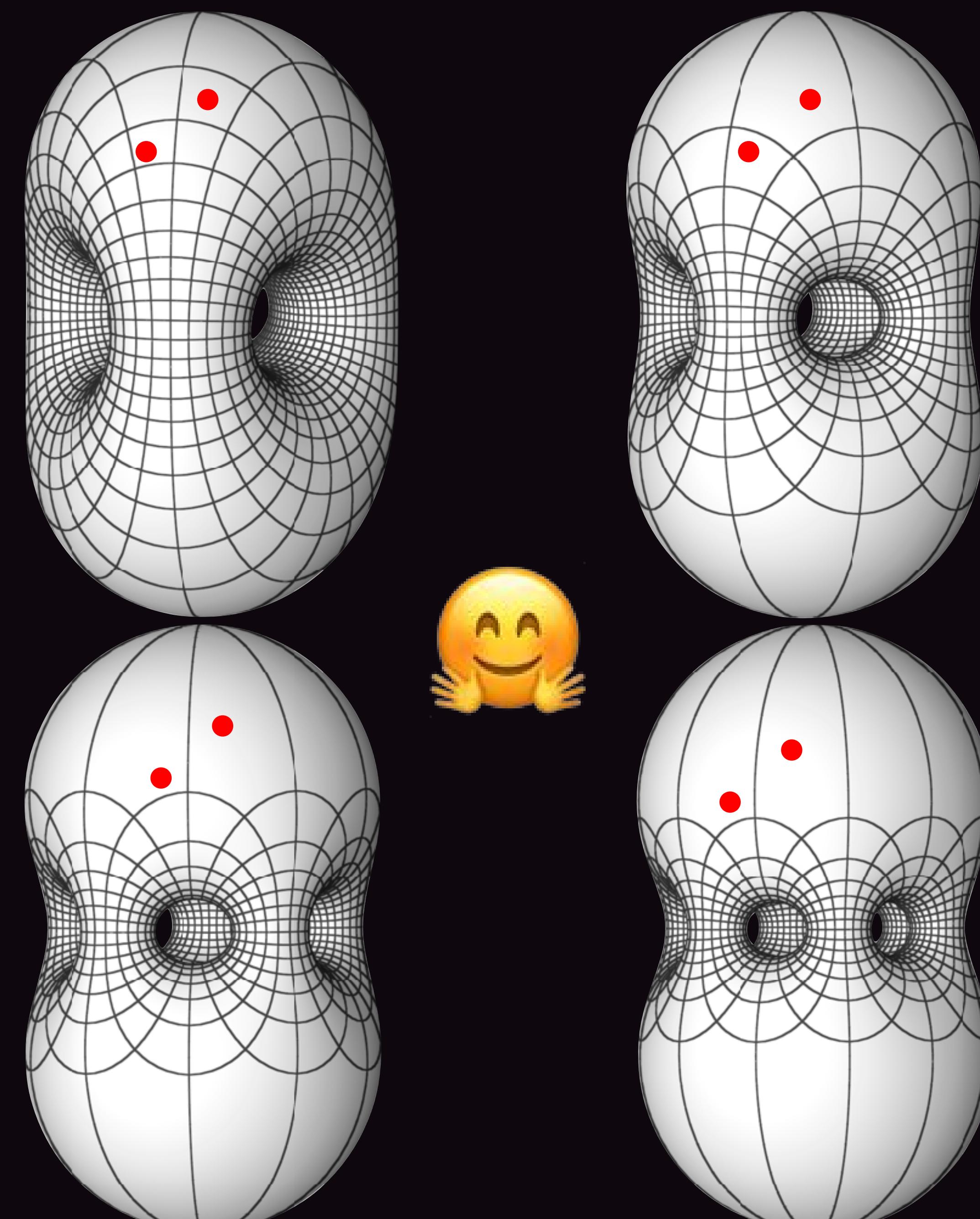
How Hard Can it Be?

Constructing CY $\subset$ Some "Nice" Ambient Space

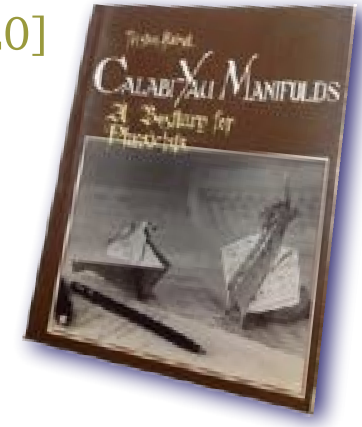
- Reduce to 0 dimensions: $\mathbb{P}^4[5] \to \mathbb{P}^3[4] \to \mathbb{P}^2[3] \to \mathbb{P}^1[2]$







[arXiv:1606.07420]



The Story so Far...

Classical Constructions

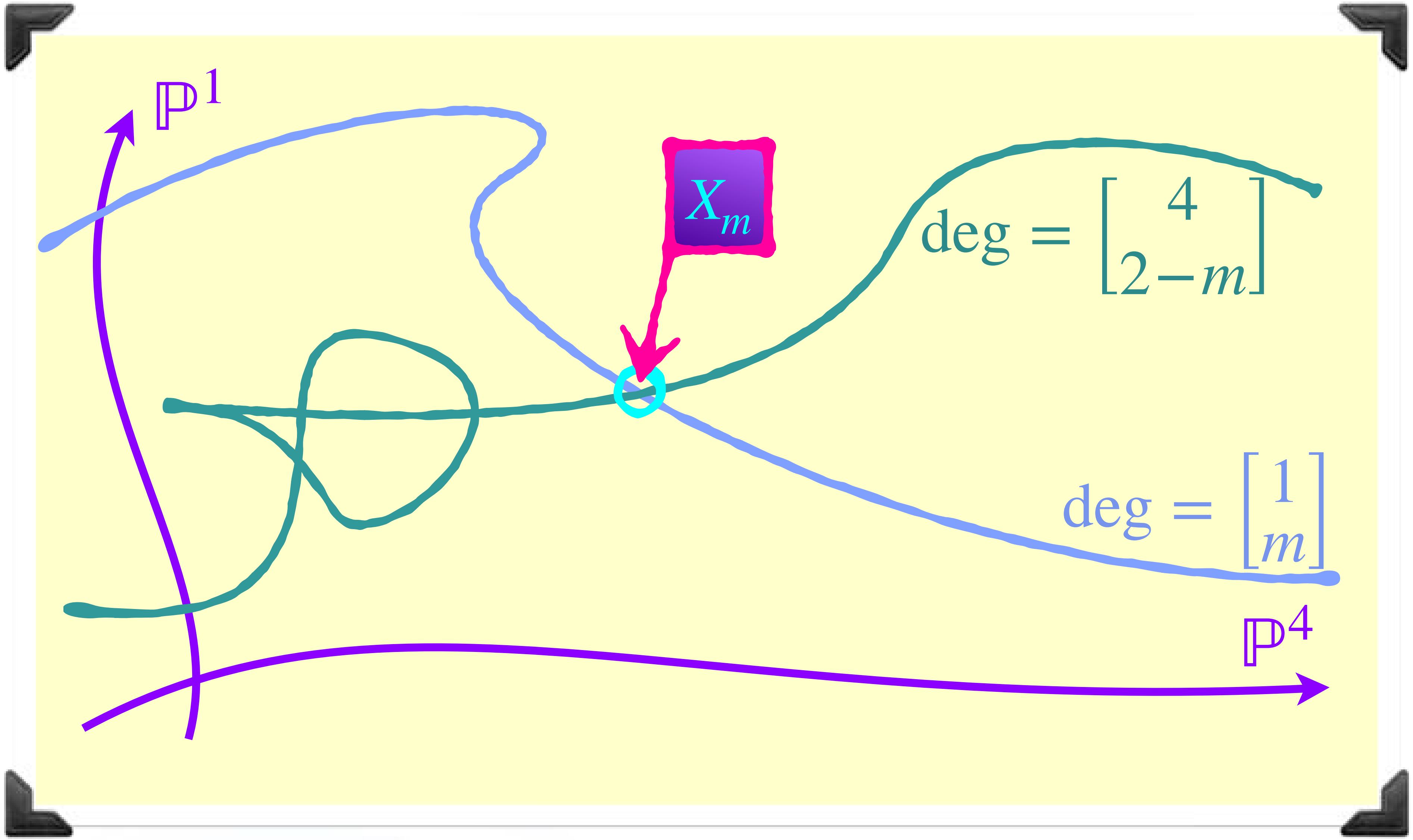
- E.g: $X_m \in \left[\begin{array}{c\|c\|c} \mathbb{P}^4 & 1 & 4 \\ \mathbb{P}^1 & m & 2-m \end{array}\right]^{(2,86)}_{-168}$

smooth$_{\mathbb{R}}$ models

$b_2 = 2 = h^{1,1}$ dim. space of Kähler classes

$\frac{1}{2}b_3 - 1 = 86 = h^{2,1}$ dim. space of cpx structures

$-168 = \chi = 2(h^{1,1} - h^{2,1})$ the Euler #



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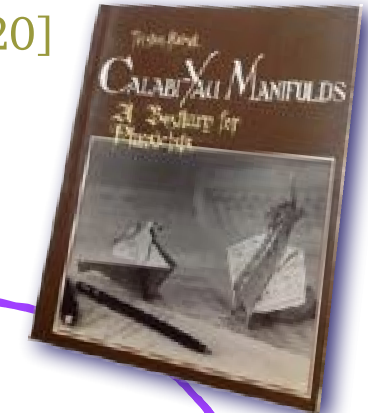


The Story so Far…

Classical Constructions

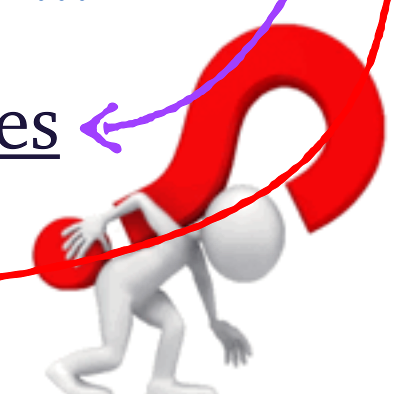
smooth$_{\mathbb{R}}$ models → *special? symplectic*

$b_2 = 2 = h^{1,1}$ dim. space of Kähler classes

$\frac{1}{2}b_3 - 1 = 86 = h^{2,1}$ dim. space of cpx structures

$-168 = \chi = 2(h^{1,1} - h^{2,1})$ the Euler #

- E.g: $X_m \in \left[\begin{array}{c||c|c} \mathbb{P}^4 & 1 & 4 \\ \mathbb{P}^1 & m & 2-m \end{array}\right]^{(2,86)}_{-168}$
  - Zero-set of $p(x,y)=0$, $\deg[p] = \binom{1}{m}$, & $q(x,y)=0$, $\deg[q] = \binom{4}{2-m}$
  - Generic $\{p=0\}\cap\{q=0\}$ smooth; $\deg_{\mathbb{P}^n}[p] + \deg_{\mathbb{P}^n}[q] = n+1 \Rightarrow c_1 = 0$
- Sequentially: $X_m \xrightarrow{q=0} \left(F_m \xrightarrow{p=0} \mathbb{P}^4\times\mathbb{P}^1\right)$ $\quad q(x,y) \overset{?}{\sim} \frac{q_0(x)}{y_0} + \frac{q_1(x)}{y_1}$
  - Chern: $c = \dfrac{(1+J_1)^5(1+J_2)^2}{(1+J_1+mJ_2)\,(1+4J_1+(2-m)J_2)}$ for F_m and X_m
  - Then: $C^{(c)}_{4-k}[(aJ_1+bJ_2)^k] = g^{(c)}_k(4b+ma)$ = *diffeomorphism* invariants
    - e.g., $\int_{F_m} c_1c_2[(aJ_1+bJ_2)] = \int_{X_m} c_2[(aJ_1+bJ_2)]$, but also $\int_{F_m} c_2[(aJ_1+bJ_2)^2]$, …
  - So, $F_m \approx_{\mathbb{R}} F_{m\,(\mathrm{mod}\,4)}$ & $X_m \approx_{\mathbb{R}} X_{m\,(\mathrm{mod}\,4)}$: 4 *diffeomorphism types*
- …so, $m = 0, 1, 2, 3 \Rightarrow \deg[q] = \binom{4}{-1}$?!



The Story so Far…

Why Haven't We Thought of This Before?

- $\deg[q] = \binom{4}{-1}$ holomorphic sections?!
  - Not *everywhere* on $\mathbb{P}^4 \times \mathbb{P}^1$ — (simple poles)
  - but yes on $F_3^{(4)} \subset \mathbb{P}^4 \times \mathbb{P}^1$ — $\geqslant 105$ of 'em!

[AAGGL:1507.03235 + BH:1606.07420]
[+ GvG:1708.00517]

$$X_m \in \left[\begin{array}{c||c|c} \mathbb{P}^4 & 1 & 4 \\ \mathbb{P}^1 & m & 2-m \end{array}\right]^{(2,86)}_{-168}$$

for $m = 3$

- How? On $F_3^{(4)}$, $q(x,y) \simeq q(x,y) + \lambda \cdot p(x,y)$ ← *equivalence class!*
  - [Hirzebruch, 1951] ⇒ $p = x_0 y_0{}^3 + x_1 y_1{}^3$ & $q = c(x)\left(\frac{x_0 y_0}{y_1{}^2} - \frac{x_1 y_1}{y_0{}^2}\right)$ $\deg[c] = \binom{3}{0}$
  - So, $q_0 = q(x,y) + \frac{\lambda\, c(x)}{(y_0 y_1)^2} p(x,y) \overset{\lambda \to -1}{=\!=\!=} c(x)\left(-2\frac{x_1 y_1}{y_0{}^2}\right)$ where $y_0 \neq 0$
  - & $q_1 = q(x,y) + \frac{\lambda\, c(x)}{(y_0 y_1)^2} p(x,y) \overset{\lambda \to 1}{=\!=\!=} c(x)\left(2\frac{x_0 y_0}{y_1{}^2}\right)$ where $y_1 \neq 0$
  - & $q_1(x,y) - q_0(x,y) = 2\frac{c(x)}{(y_0 y_1)^2} p(x,y) = 0$, on $F_3 := \{p(x,y) = 0\}$

Wu-Yang monopole

  - [GvG, 1708.00517] *scheme-th*. "*generalized* complete intersections"

Reverse-engineered: Mayer-Vietoris sequence & "patching" of the two charts

$p_0^{-1}(0) \cap \mathfrak{s}^{-1}(0)$ is smooth
$\mathrm{d}p_0(x,y) \wedge \mathrm{d}\mathfrak{s}(x,y) \neq 0$

Fusing Fugue

…in well-tempered counterpoint

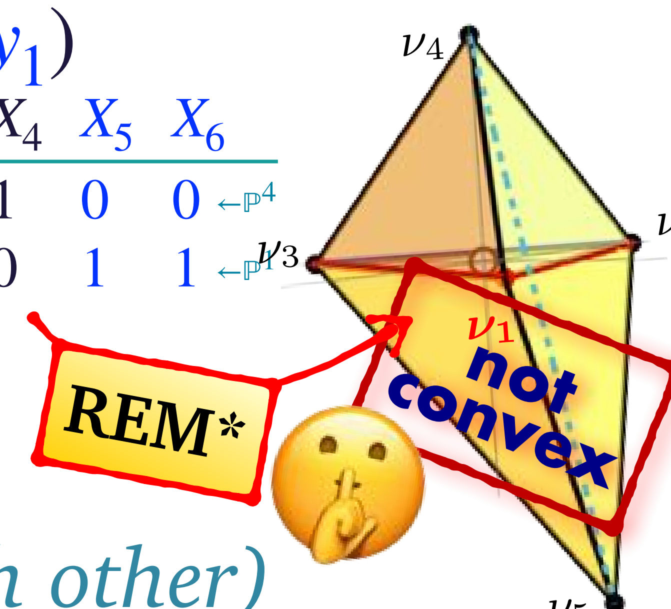
[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139] +more

- For $\Big\{\underbrace{x_0 y_0{}^m + x_1 y_1{}^m}_{:=p(x,y;0)} = -\sum_{a=2}^{n}\sum_{\ell=1}^{m-1} \epsilon_{a,\ell}\, x_a\, y_0{}^{m-\ell} y_1{}^{\ell} \Big\} = F^{(n)}_{m;\epsilon} \in \begin{bmatrix} \mathbb{P}^n \\ \mathbb{P}^1 \end{bmatrix}\!\!\begin{Vmatrix} 1 \\ m \end{Vmatrix}$

  even $p(x,y;0)$ is transverse, $p^{-1}(0)$ is smooth

  - The central ($\epsilon = 0$) member of the family is a Hirzebruch scroll F_m :
  - Directrix: $\mathfrak{s}(x,y) := \Big(\frac{x_0}{y_1{}^m} - \frac{x_1}{y_0{}^m}\Big) + \frac{\lambda}{(y_0 y_1)^m}[x_0 y_0{}^m + x_1 y_1{}^m]$ degree $\binom{1}{-m}$
- On $F^{(n)}_m$: $p(x,y;0) = x_0 y_0{}^m + x_1 y_1{}^m = 0 \Rightarrow x_0 = -x_1 (y_1/y_0)^m$, @ $y_0 \neq 0$
  - So, $x_1 \to X_1 = \mathfrak{s}(x,y)$ & $(X_i,\ i = 2,\cdots) = (x_2,\cdots,x_n;\, y_0, y_1)$
  - Key: $\det\left[\frac{\partial(p(x,y), \mathfrak{s}(x,y), x_2, \cdots; y_0, y_1)}{\partial(x_0, x_1, x_2, \cdots; y_0, y_1)}\right] = \text{const.}$
  - $\mathbb{P}^4 \times \mathbb{P}^1$ bi-degree → toric $(\mathbb{C}^\times)^2$-action:

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	←$\mathbb{P}^4$
$-m$	0	0	0	1	1	←$\mathbb{P}^1$



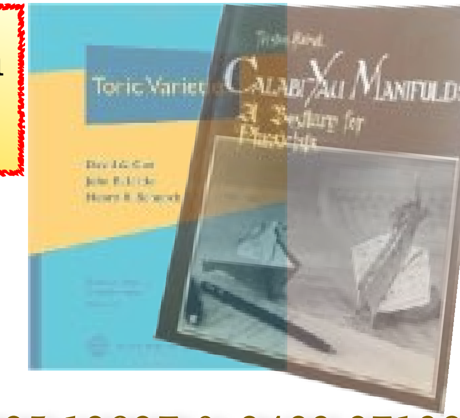
- *And the rest of the $\epsilon_{a,\ell}$-deformation family?*
  (where smooth models are all diffeomorphic to each other)

*Reverse-Engineered (Toric) Model

Fusing Fugue

…with a meandering motif

$p_0^{-1}(0) \cap \mathfrak{s}^{-1}(0)$ **is smooth**
$\mathrm{d}p_0(x,y) \wedge \mathrm{d}\mathfrak{s}(x,y) \neq 0$



[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139]
+more

- **Construction 2.1** *Given a degree-$\binom{1}{m}$ hypersurface $\{p_{\vec\epsilon}(x,y)\,0\} \subset \mathbb{P}^n \times \mathbb{P}^1$ as in (2.2), construct*

$$\deg = \binom{1}{m-r_0-r_1}: \quad \mathfrak{s}_{\vec\epsilon}(x,y;\lambda) := \operatorname*{Flip}_{y_0}\Big[\frac{1}{y_0{}^{r_0}\, y_1{}^{r_1}}\, p_{\vec\epsilon}(x,y)\Big] \pmod{p_{\vec\epsilon}(x,y)}, \qquad \left[\begin{matrix}\mathbb{P}^n\\ \mathbb{P}^1\end{matrix}\middle\|\begin{matrix}1\\ m\end{matrix}\right]$$

progressively decreasing $r_0+r_1=2m, 2m-1, \cdots$, and keeping only Laurent polynomials containing both y_0- and y_1-denominators but no y_0,y_1-mixed ones. The "Flip_{y_i}" operator changes the relative sign of the rational monomials with y_i-denominators. For algebraically independent such sections, restrict to a subset with maximally negative degrees that are not overall (y_0,y_1)-multiples of each other.

- E.g.: $m=2$

```
p0 = x0 y0^2 + x1 y1^2; ep[α_] := Table[1/(y0^(α-i) y1^i), {i, 0, α}]; Expand /@ (p0 {ep[5], ep[4], ep[3]})
```

$$\Big\{\Big\{\frac{x_0}{y_0^3}+\frac{x_1 y_1^2}{y_0^5},\ \frac{x_0}{y_0^2 y_1}+\frac{x_1 y_1}{y_0^4},\ \frac{x_1}{y_0^3}+\frac{x_0}{y_0 y_1^2},\ \frac{x_0}{y_1^3}+\frac{x_1}{y_0^2 y_1},\ \frac{x_0 y_0}{y_1^4}+\frac{x_1}{y_0 y_1^2},\ \frac{x_0 y_0^2}{y_1^5}+\frac{x_1}{y_1^3}\Big\},$$

$\cdot\, y_1, \quad \cdot\, y_0$

$$\Big\{\frac{x_0}{y_0^2}+\frac{x_1 y_1^2}{y_0^4},\ \frac{x_0}{y_0 y_1}+\frac{x_1 y_1}{y_0^3},\ \frac{x_1}{y_0^2}+\frac{x_0}{y_1^2},\ \frac{x_0 y_0}{y_1^3}+\frac{x_1}{y_0 y_1},\ \frac{x_0 y_0^2}{y_1^4}+\frac{x_1}{y_1^2}\Big\},\ \Big\{\frac{x_0}{y_0}+\frac{x_1 y_1^2}{y_0^3},\ \frac{x_0}{y_1}+\frac{x_1 y_1}{y_0^2},\ \frac{x_1}{y_0}+\frac{x_0 y_0}{y_1^2},\ \frac{x_0 y_0^2}{y_1^3}+\frac{x_1}{y_1}\Big\}\Big\}$$

  - finds $\mathfrak{s}(x,y) = \Big(\frac{x_0}{y_1{}^2} - \frac{x_1}{y_0{}^2}\Big) \bmod \big(x_0 y_0{}^2 + x_1 y_1{}^2\big)$; $\deg = \binom{1}{-2}$, $[\mathfrak{s}^{-1}(0)] = [J_1] - 2[J_2]$.

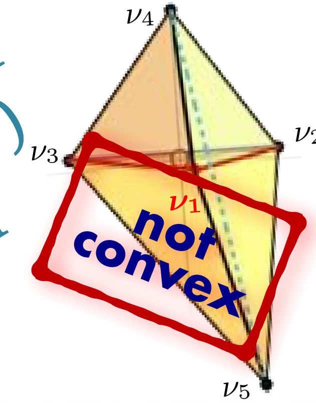
THE **exceptional section**

THE **exceptional curve** $[S]^2 = -1$ **in** $F_2^{(2)}$

Fusing Fugue

…in well-tempered counterpoint

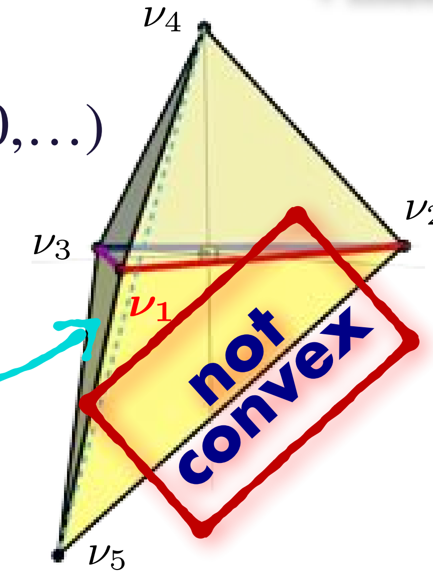
central ($\epsilon_{a,\ell}=0$) model



[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139] +more

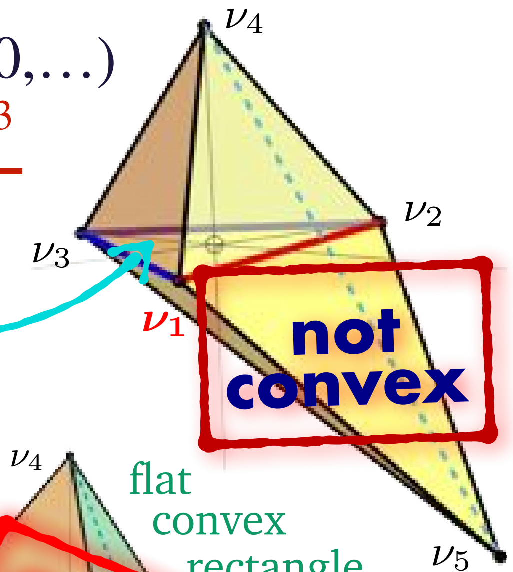
- Deform: $p_1(x,y)=x_0y_0{}^5+x_1y_1{}^5+x_2y_0{}^4y_1$ toric $F^{(n)}_{(4,1,0,\dots)}$
  - Now: $\mathfrak{s}_{1,1}(x,y)=\frac{x_0y_0}{y_1{}^5}+\frac{x_2}{y_1{}^4}-\frac{x_1}{y_1{}^4}$ & $\mathfrak{s}_{1,2}(x,y)=\frac{x_0}{y_1}-\frac{x_2}{y_0}-\frac{x_1y_1{}^4}{y_0{}^5}$
  - & $\det\left[\frac{\partial(p_1,\mathfrak{s}_{1,1},\mathfrak{s}_{1,2},x_3,\cdots;y_0,y_1)}{\partial(x_0,x_1,x_2,x_3,\cdots;y_0,y_1)}\right]=\text{const.}$

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	←$\mathbb{P}^4$
−4	−1	0	0	1	1	←$\mathbb{P}^1$

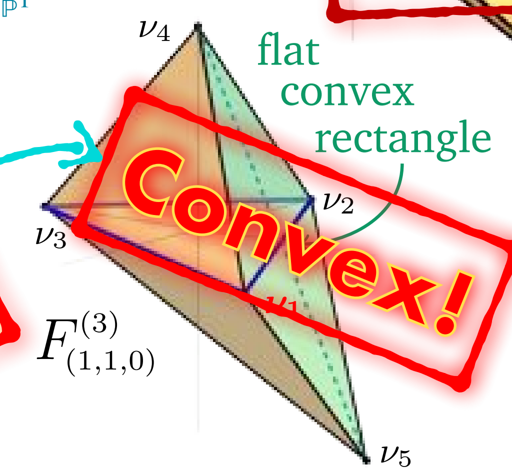


- Deform: $p_2(x,y)=x_0y_0{}^5+x_1y_1{}^5+x_2y_0{}^3y_1{}^2$ toric $F^{(n)}_{(3,2,0,\dots)}$
  - Now: $\mathfrak{s}_{2,1}(x,y)=\frac{x_0y_0{}^2}{y_1{}^5}+\frac{x_2}{y_1{}^3}-\frac{x_1}{y_1{}^3}$ & $\mathfrak{s}_{2,2}(x,y)=\frac{x_0}{y_1{}^2}-\frac{x_2}{y_0{}^2}-\frac{x_1y_1{}^3}{y_0{}^5}$
  - & $\det\left[\frac{\partial(p_2,\mathfrak{s}_{2,1},\mathfrak{s}_{2,2},x_3,\cdots;y_0,y_1)}{\partial(x_0,x_1,x_2,x_3,\cdots;y_0,y_1)}\right]=\text{const.}$

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	←$\mathbb{P}^4$
−3	−2	0	0	1	1	←$\mathbb{P}^1$

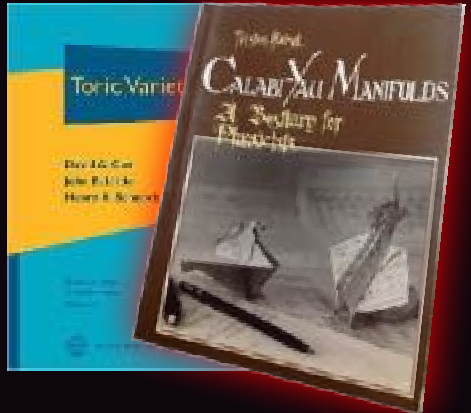


- … and $p_3(x,y)=x_0y_0{}^5+x_1y_1{}^5+x_2y_0{}^3y_1{}^2+x_3y_0{}^2y_1{}^3$
  - → toric $F^{(n)}_{(2,2,1,\cdots)}$ for $n=3$, $F^{(3)}_{(2,2,1)}\approx_{\mathbb{R}}F^{(3)}_{(1,1,0)}$ [∼Segre] Fano!



Fusing Fugue

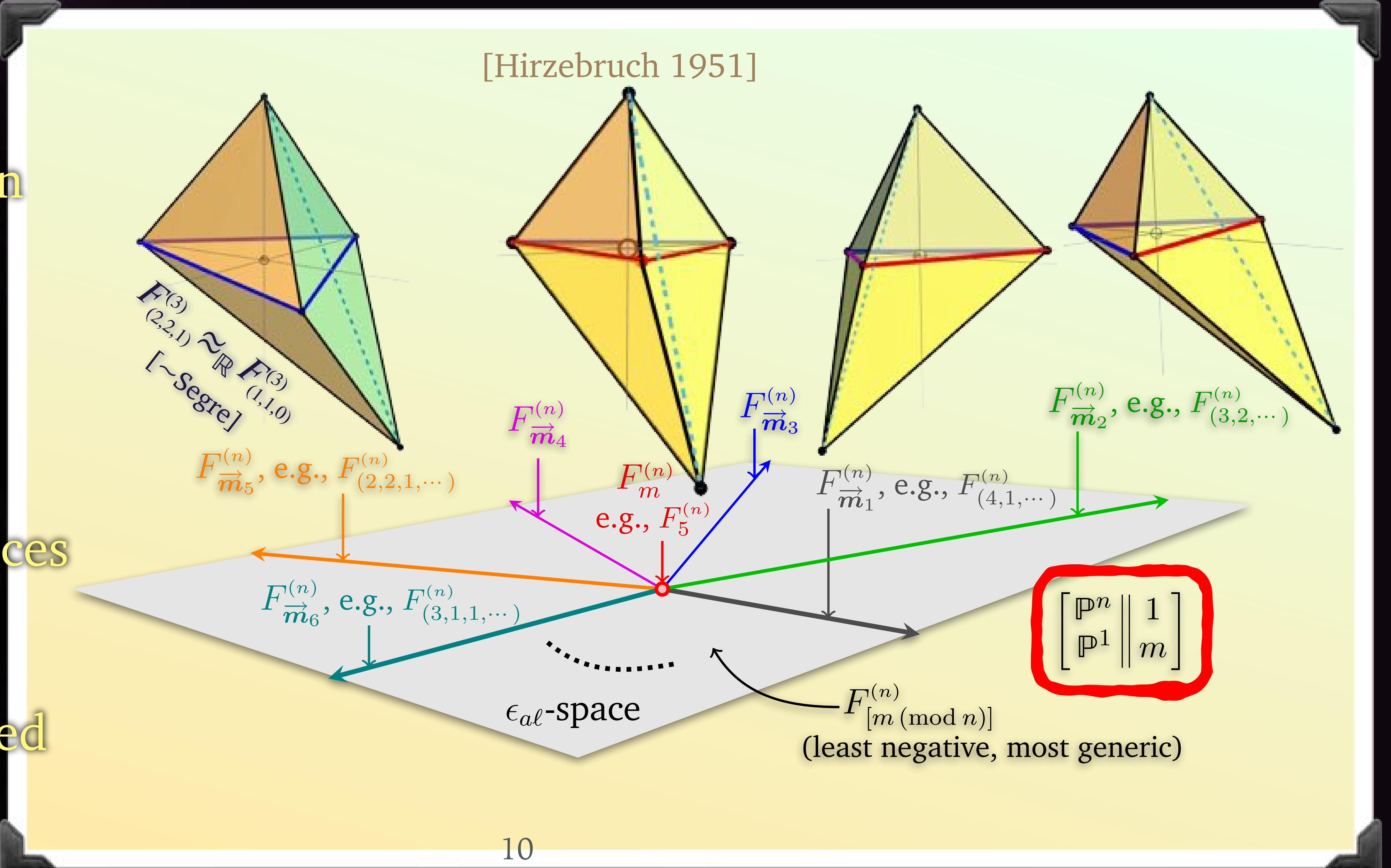
…in well-tempered counterpoint



[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139]
+more

- A deformation *family picture*:

- One complete intersection
- to “rule them all” (toric varieties)
- …and CY hypersurfaces threin
- …and *web* of connected GLSMs



Meromorphic March

…back to the medial motif

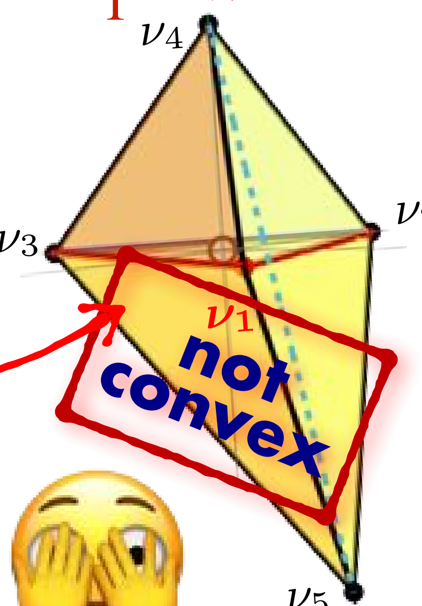
[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139] +more

- On $F_m^{(n)}$: $p(x,y;0) = x_0 y_0{}^m + x_1 y_1{}^m = 0 \Rightarrow x_0 = -x_1 (y_1/y_0)^m$ & $x_1 \to X_1 = \mathfrak{s}$
  - & $(X_i,\ i=2,\cdots,n+2) = (x_2,\cdots,x_n; y_0, y_1)$
  - $\mathbb{P}^4 \times \mathbb{P}^1$ bi-degree → toric $(\mathbb{C}^\times)^2$-action:

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	$\leftarrow \mathbb{P}^4$
$-m$	0	0	0	1	1	$\leftarrow \mathbb{P}^1$

  - BTW, $\det\left[\frac{\partial(p(x,y), \mathfrak{s}(x,y), x_2, \cdots; y_0, y_1)}{\partial(x_0, x_1, x_2, \cdots; y_0, y_1)}\right] = \text{const.}$
  - CY: need $\deg[f(X)] = \binom{4}{2-m}$; $\deg[X_1 X_{5,6}^m] = \binom{1}{0} = \deg[X_{2,3,4}]$

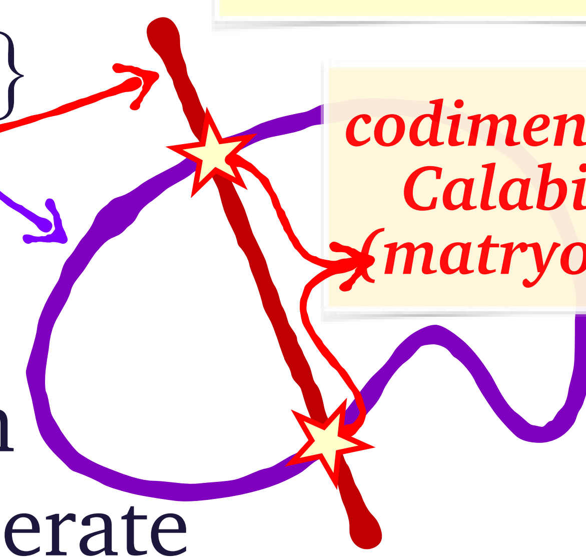
REM*



- $f(X) = X_1{}^4 X_{5,6}^{2+3m} \oplus X_1{}^3 X_{2,3,4} X_{5,6}^{2+2m} \cdots \oplus X_1 X_{2,3,4}^3 X_{5,6}^2 \oplus X_{2,3,4}^4 X_{5,6}^{2-m}$

standard wisdom

  - $m > 2$, $\{f(X)=0\} = \{X_1 = 0\} \cup \{\oplus_k X_1{}^k X_{2,3,4}^2 X_{5,6}^{2+km} = 0\}$
  - $\{f(X)=0\}^\sharp = \{X_1 = 0\} \cap \{\oplus_{k=0}^3 X_1{}^k X_{2,3,4}^{4-k} X_{5,6}^{2+km} = 0\}$
  - Standard wisdom: these are un-smoothable.



codimension-2 Calabi-Yau (matryoshka)

Tyurin degenerate

*Reverse-Engineered Model

Meromorphic March

…back to the medial motif

[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139]
+more

- On $F_m^{(n)}$: $x_0 y_0{}^m + x_1 y_1{}^m = 0 \;\Rightarrow\; x_0 = -x_1 (y_1/y_0)^m$ & $x_1 \to X_1 = \mathfrak{s}$
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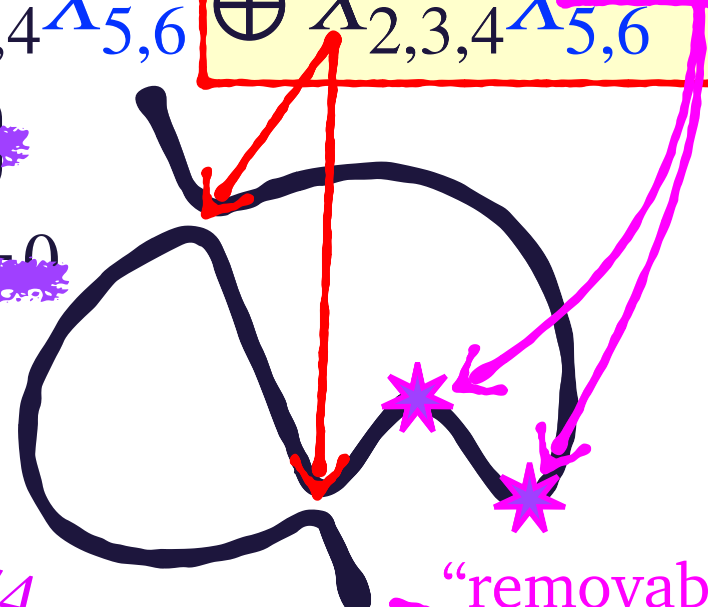
- $f(X) = X_1{}^4 X_{5,6}^{2+3m} \oplus X_1{}^3 X_{2,3,4} X_{5,6}^{2+2m} \cdots \oplus X_1 X_{2,3,4}^3 X_{5,6}^2 \oplus X_{2,3,4}^4 X_{5,6}^{2-m}$
  - $m>2$, $\{f(X)=0\}$ ~~$= \{X_1 = 0\} \cup \{\oplus_k X_1^k X_{2,3,4}^2 X_{5,6}^{2+km} = 0\}$~~
  - ~~$\{f(X)=0\}^\sharp = \{X_1=0\} \cap \{\oplus_k X_1^k X_{2,3,4}^2 X_{5,6}^{2+km} = 0\}$: $P_\mu = 0$~~
  - Embrace the Laurent terms = transverse
  - "Intrinsic limit" (L'Hôpital-"repaired")
    → smooth (*pre?*)complex spaces



$b_2 = 2,\ b_3 = 174$
$(h^{2,1} = 86)$

"removable" singularity

Mirror Minuet

& Non-Convex Mirrors

—2D *Proof-of-Concept*—

[… 2205.12827 & 2403.07139]
+more

$m=3$

- $X_1{}^2 X_2{}^0 (X_3 \oplus X_4)^{2+1m} \oplus X_1{}^1 X_2{}^1 (X_3 \oplus X_4)^{2+0m}$

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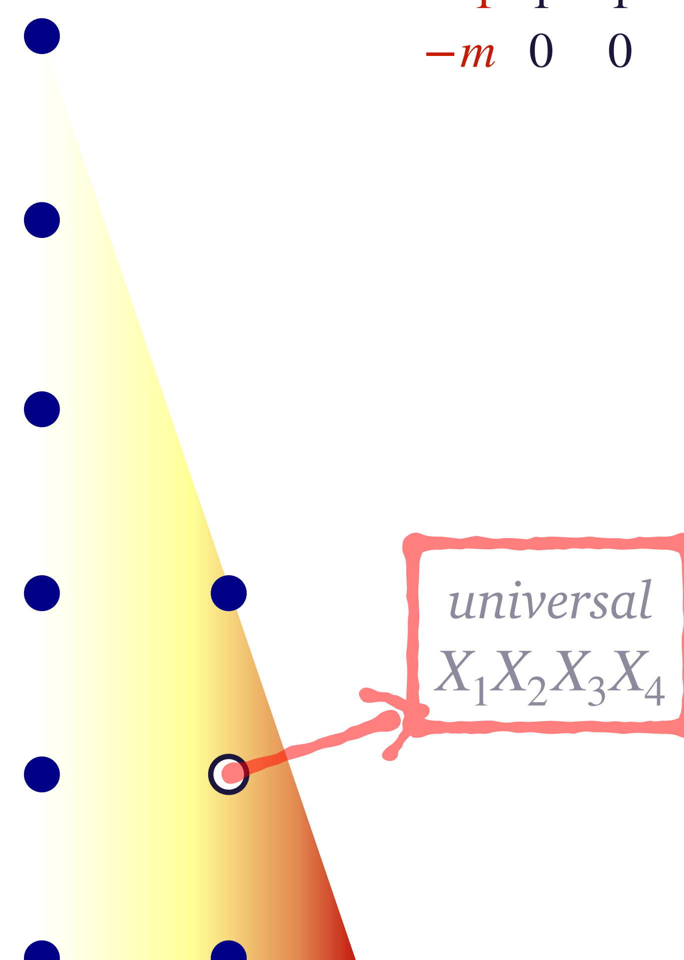

…how Gell-Mann felt,
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Mirror Minuet

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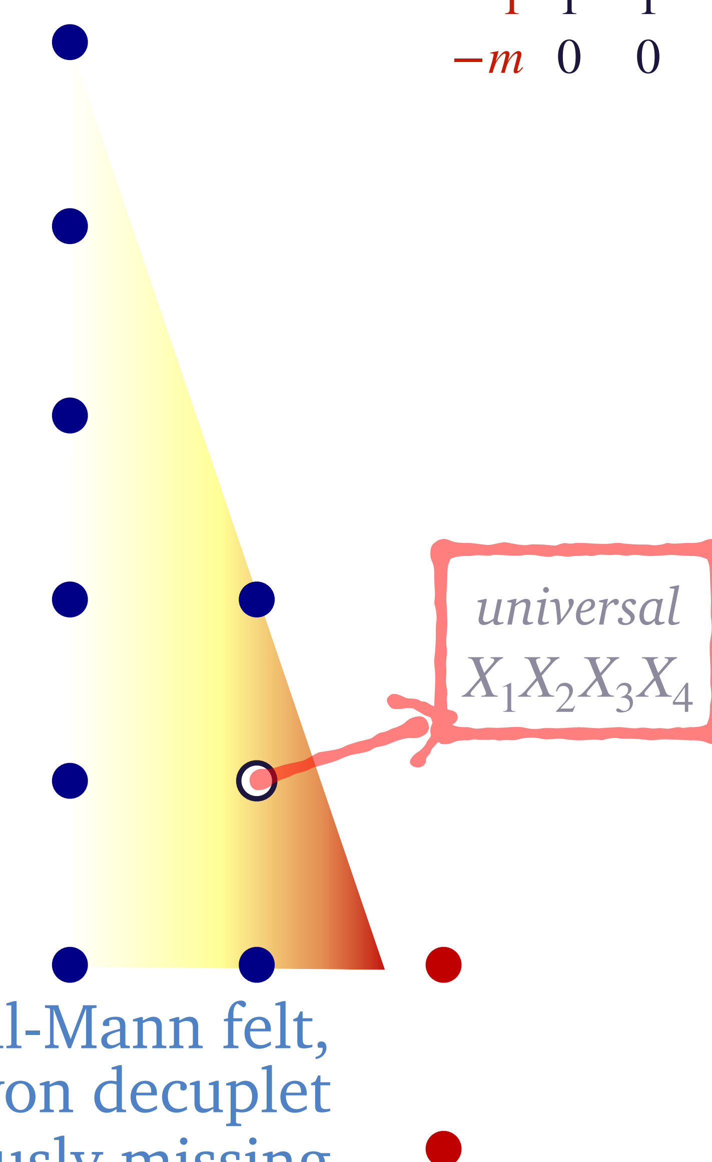

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Mirror Minuet

& Non-Convex Mirrors —2D *Proof-of-Concept*—

…2205.12827 & 2403.07139] +more

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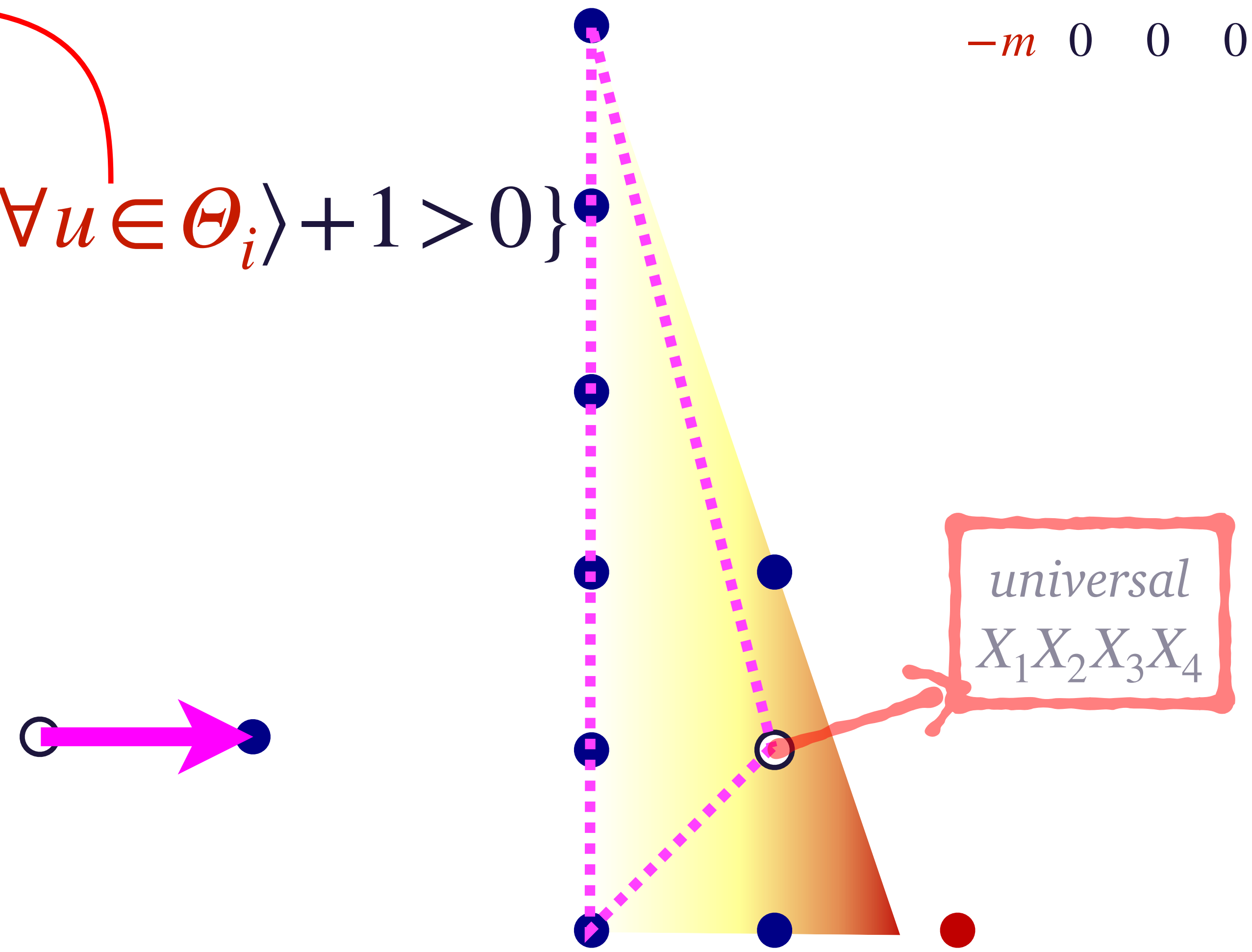

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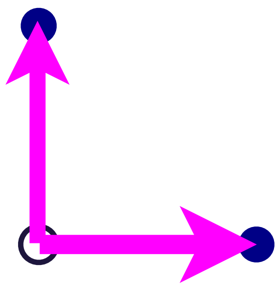

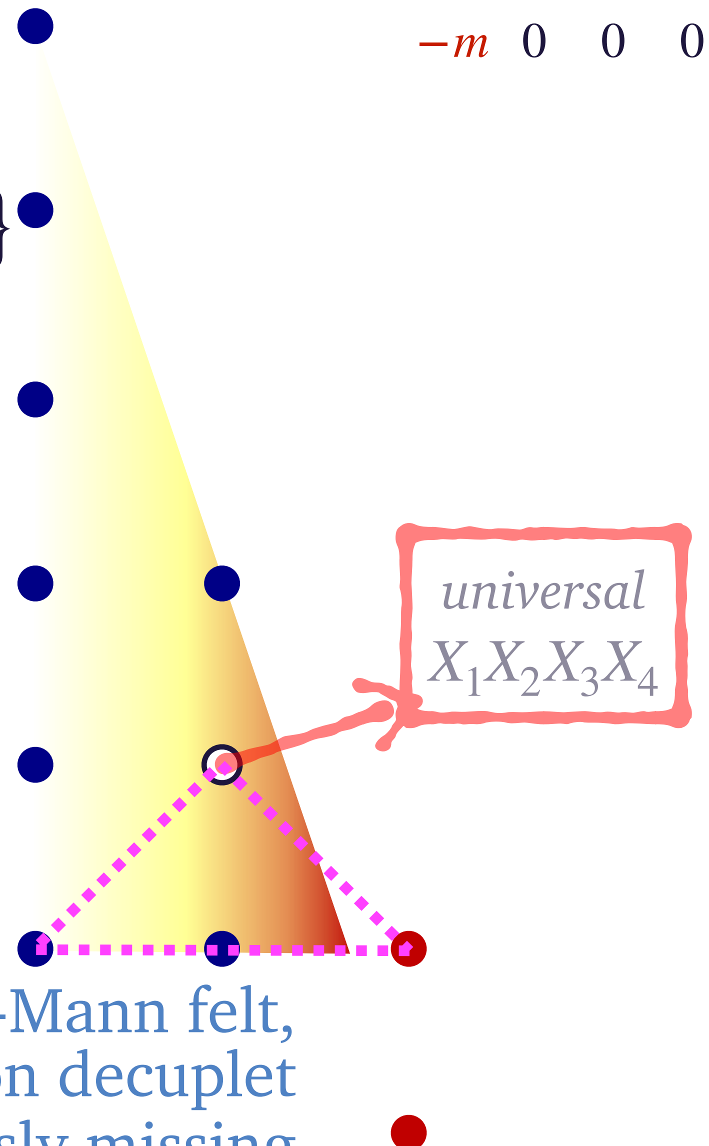

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& Non-Convex Mirrors

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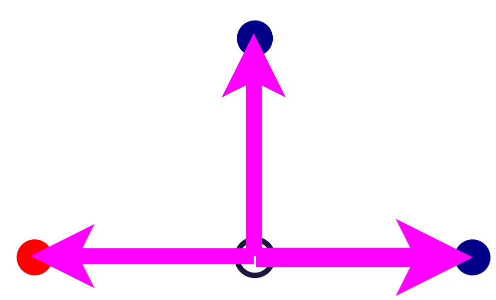

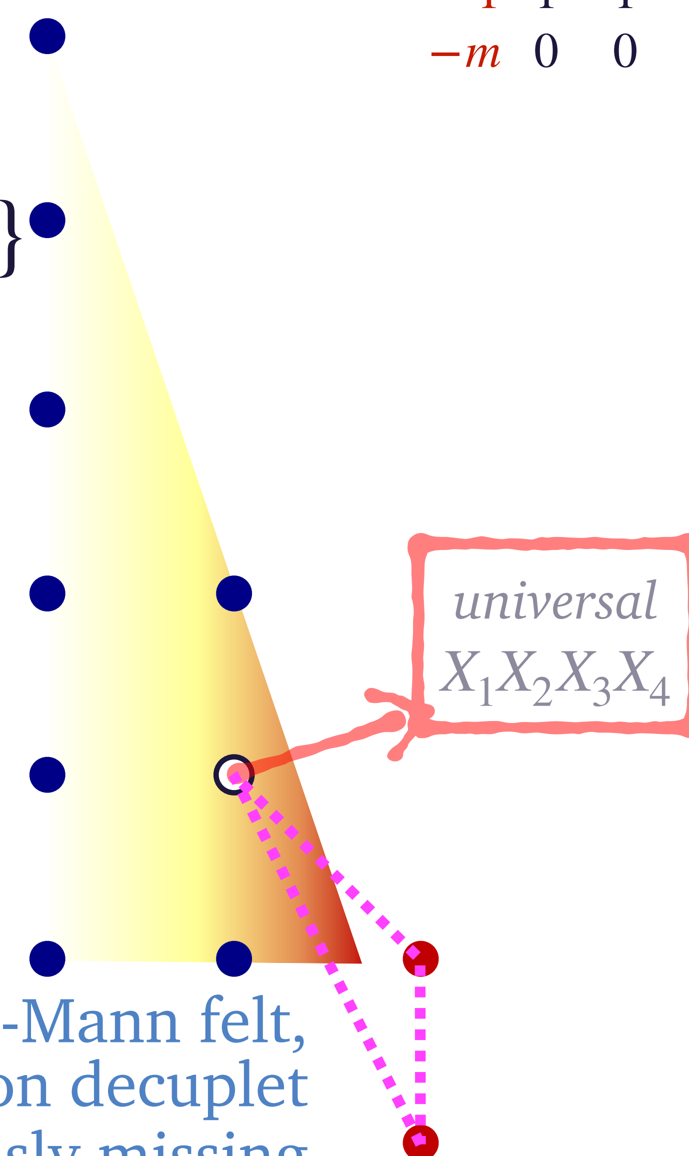

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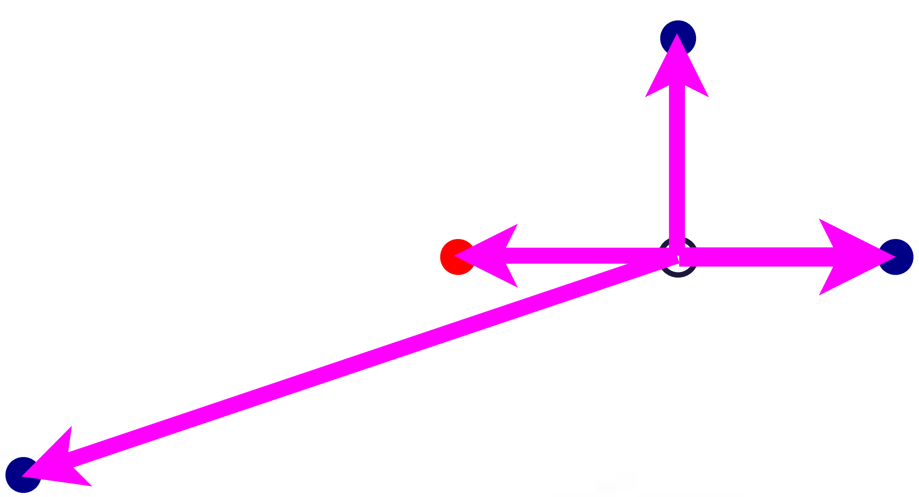

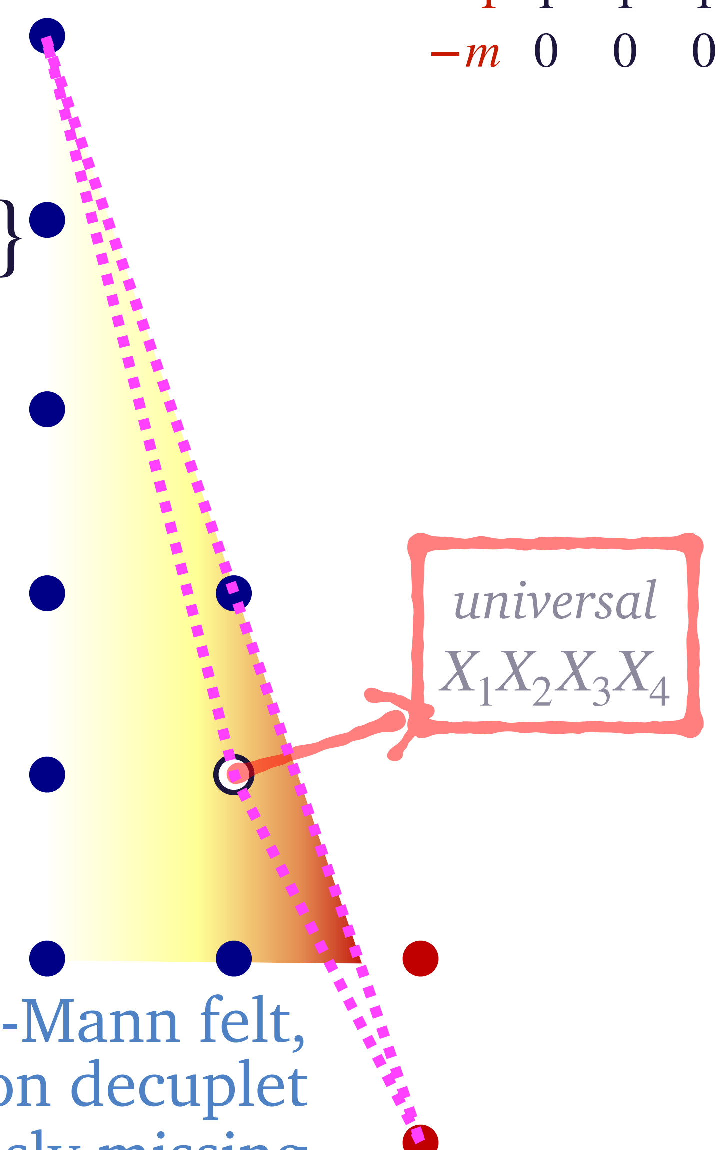

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& Non-Convex Mirrors —2D Proof-of-Concept—

2205.12827 & 2403.07139]
+more

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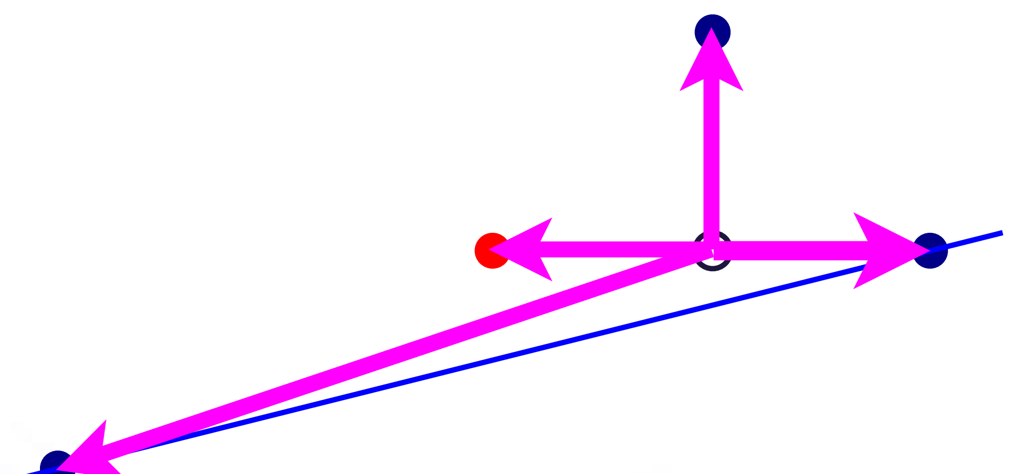

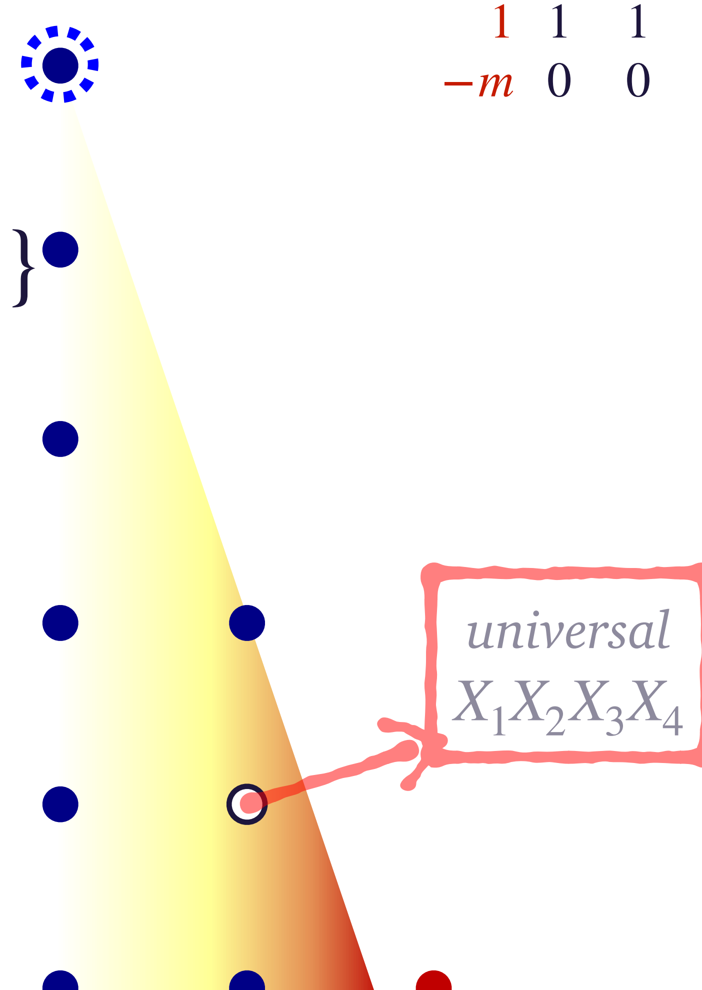

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& Non-Convex Mirrors

$m=3$ —2D *Proof-of-Concept*— [… 2205.12827 & 2403.07139] +more

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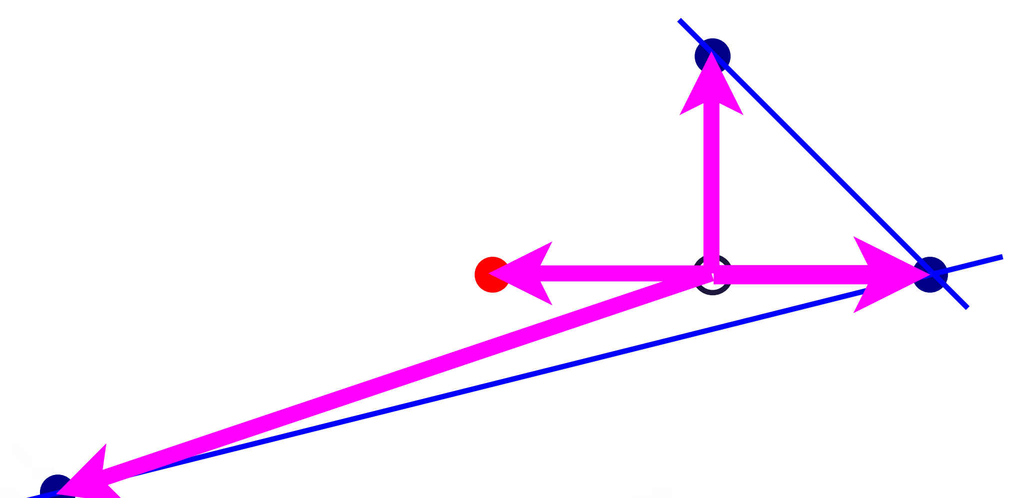

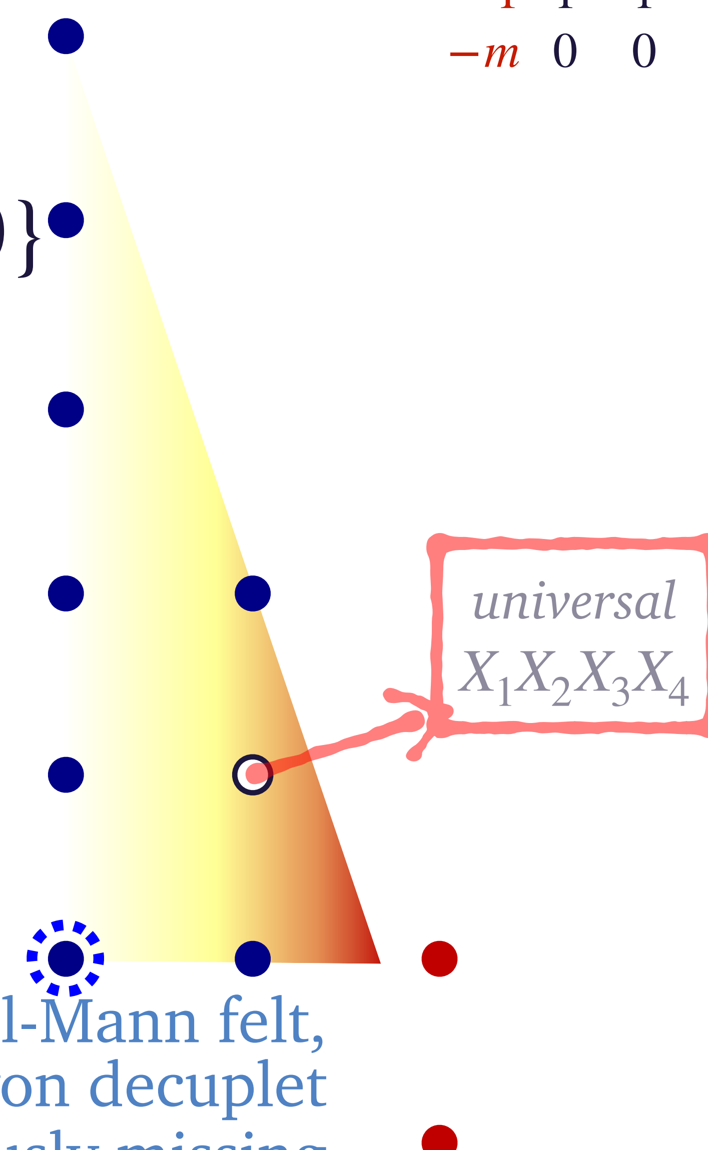

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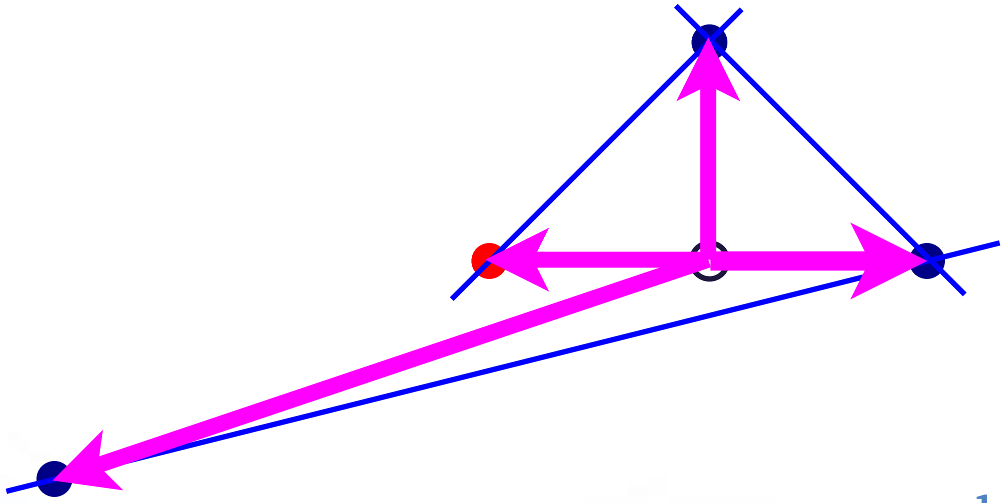

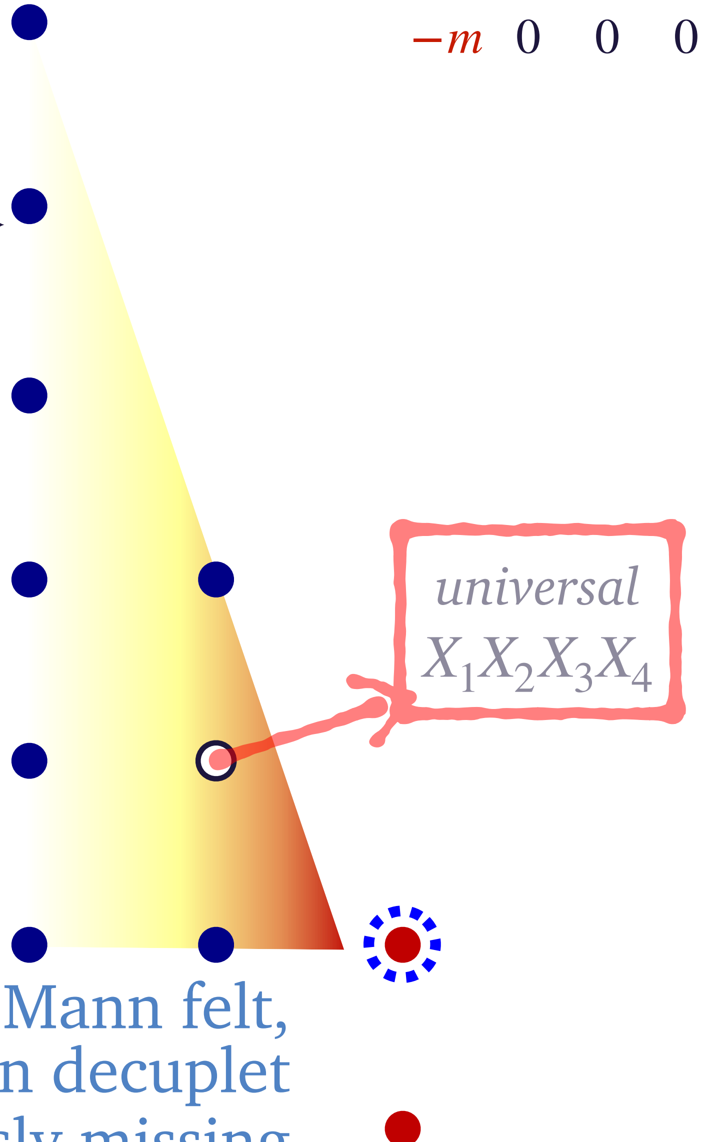

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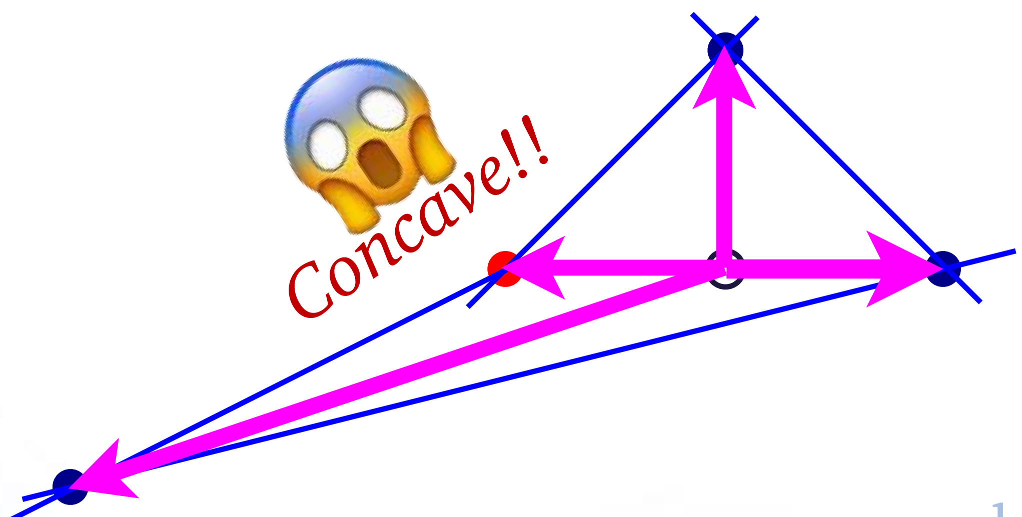

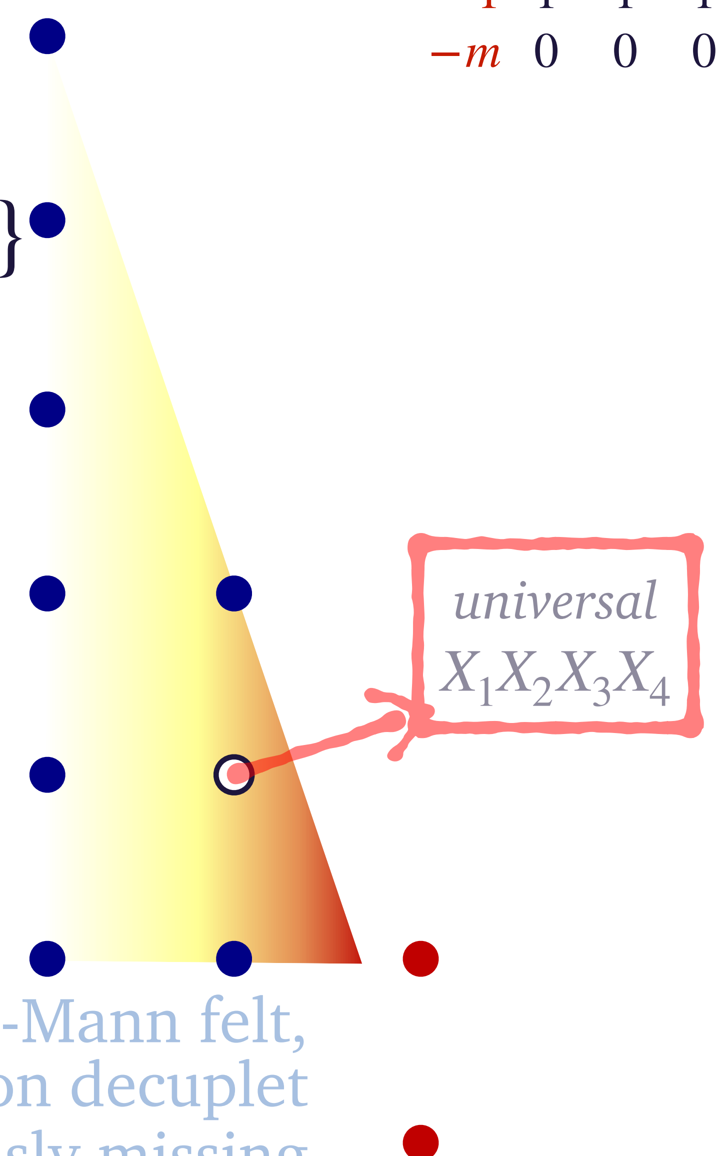

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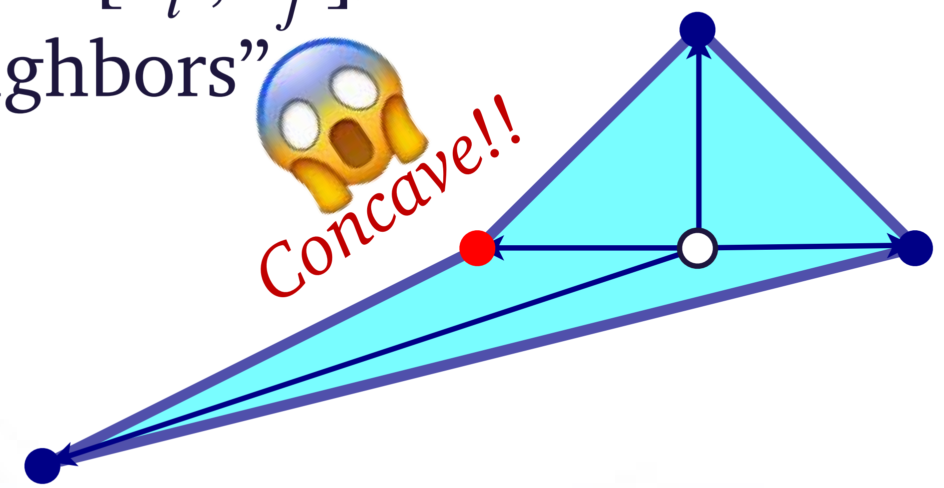

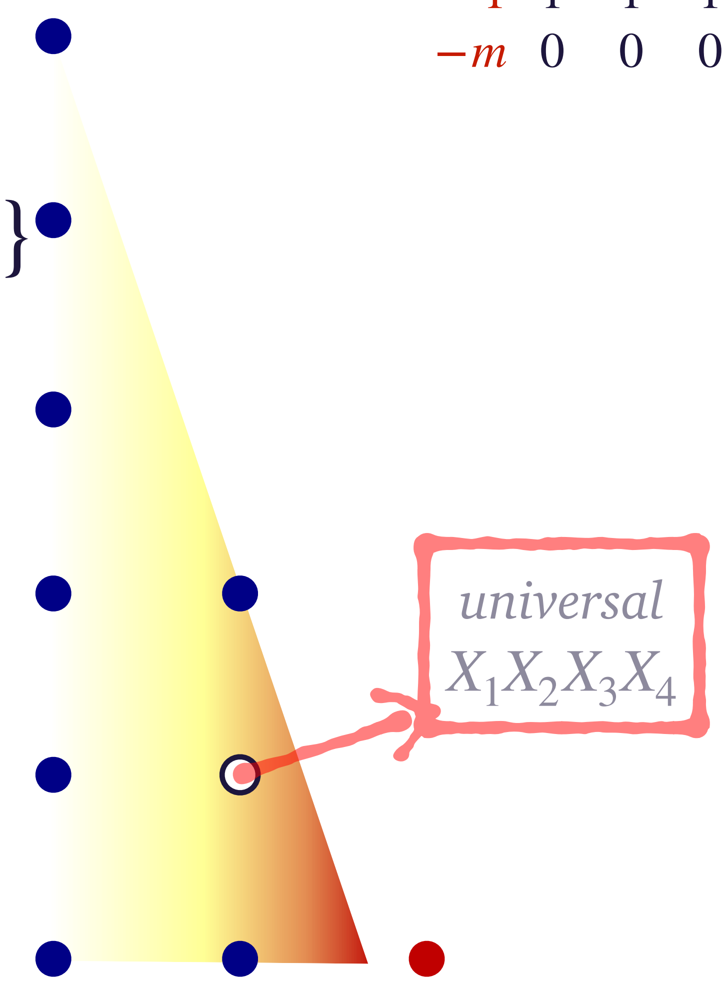

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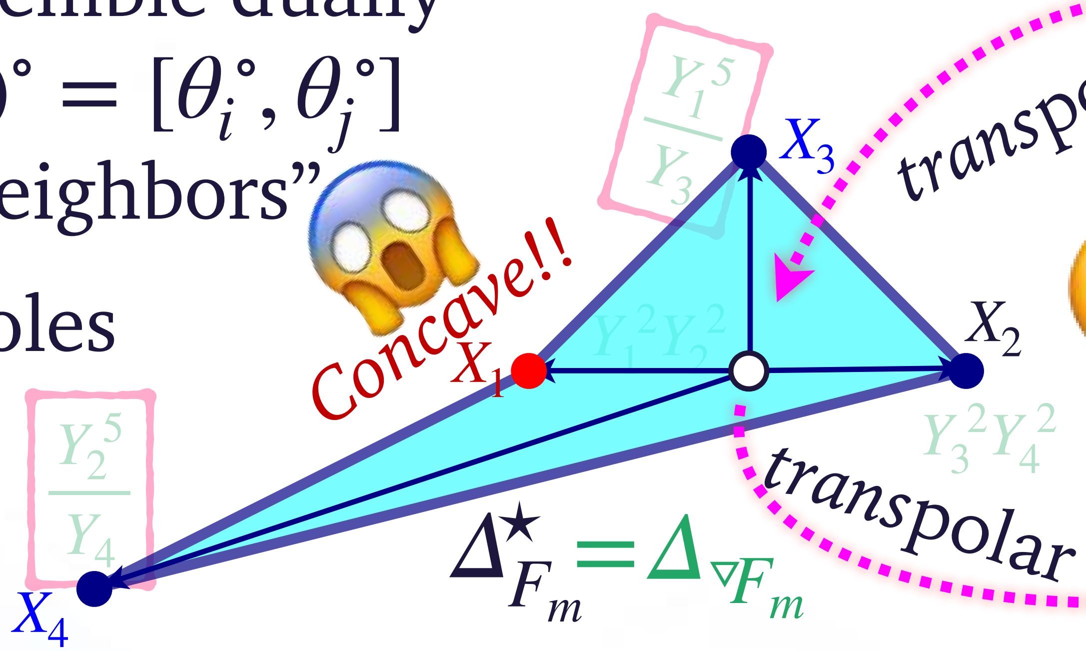
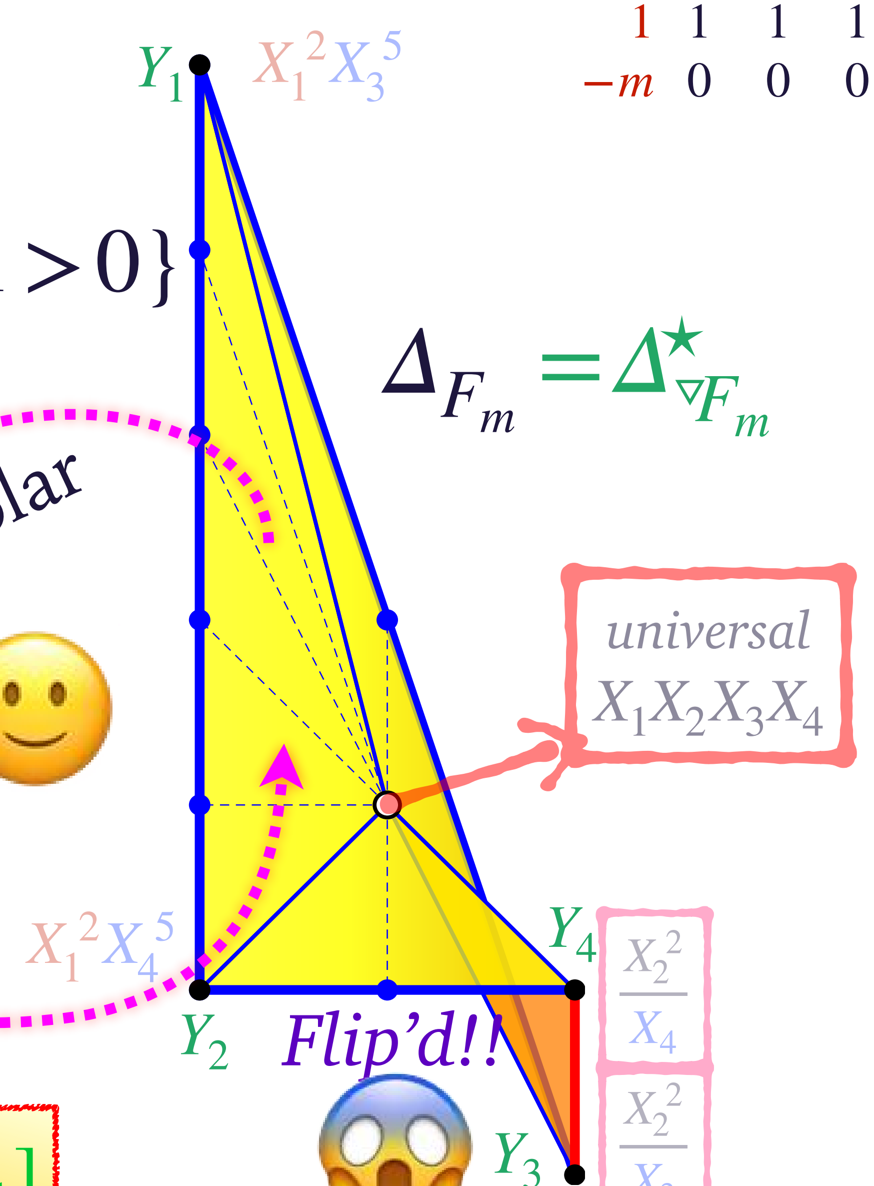
  - Swap roles $X_i \leftrightarrow Y_I$





$F_m[c_1] \overset{\text{MM}}{\longleftrightarrow} {}^\nabla F_m[c_1]$

Mirror Minuet

& Non-Convex Mirrors

$m=3$ —2D Proof-of-Concept— [… 2205.12827 & 2403.07139] +more

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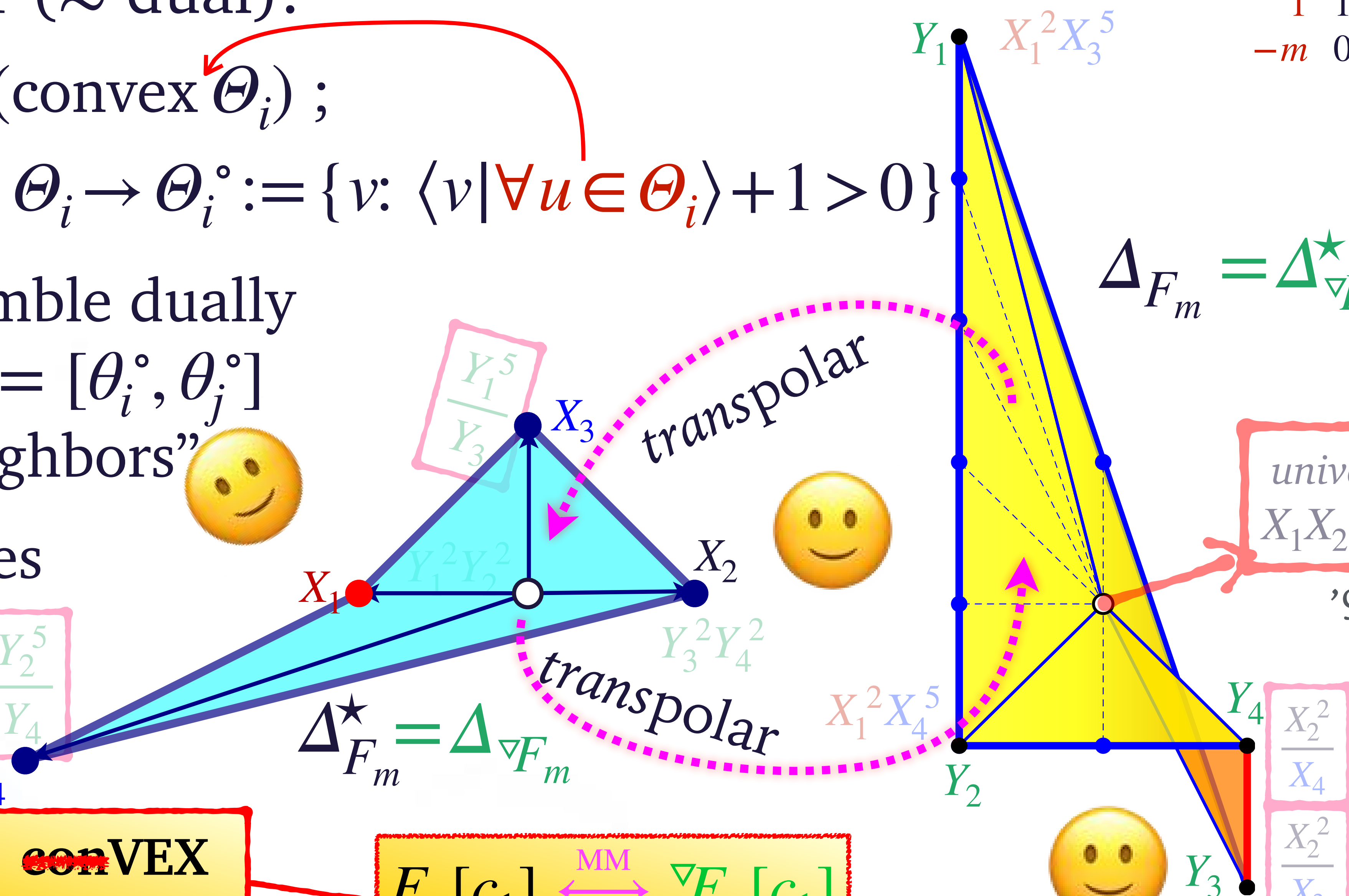


(pre)complex algebraic geometry

~~con~~VEX polytopes

$F_m[c_1] \overset{\text{MM}}{\longleftrightarrow} {}^{\nabla}F_m[c_1]$

'92: Khovanskii +Pukhlikov
'93: Karshon +Tolman
'99: Hattori +Masuda
+lots of

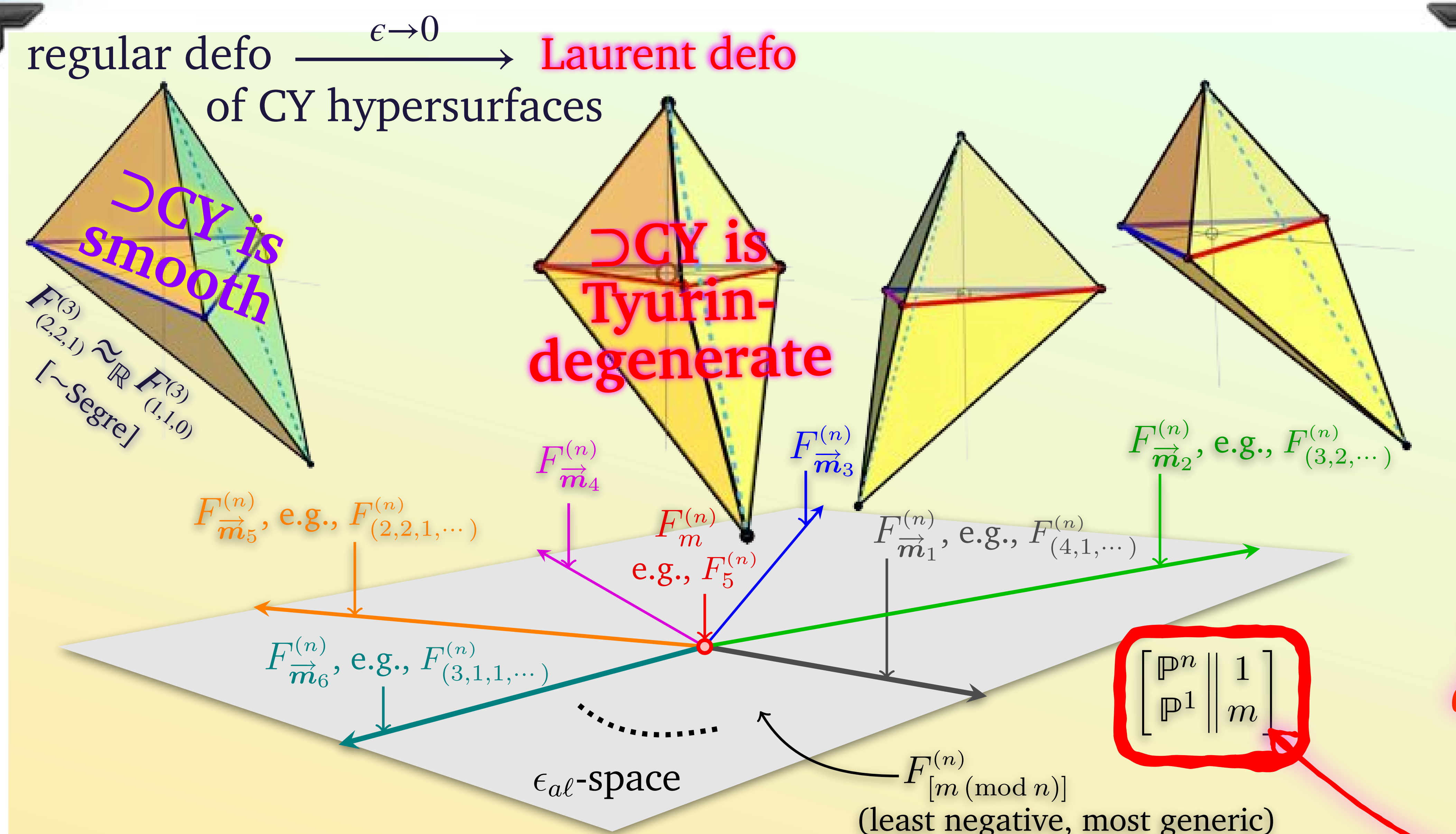
(pre)symplectic geometry

Mirror Minuet

Summary

—*n*D *Proof-of-Concept*— […, 2205.12827 & 2403.07139] +more

- CY(n–1)-folds in Hirzebruch n-folds, $X_m^{(n-1)} \in F_m^{(n)}[c_1]$



A deformation family picture

each distinct $F_m^{(n)}$ harbors multiple transp. mirror models

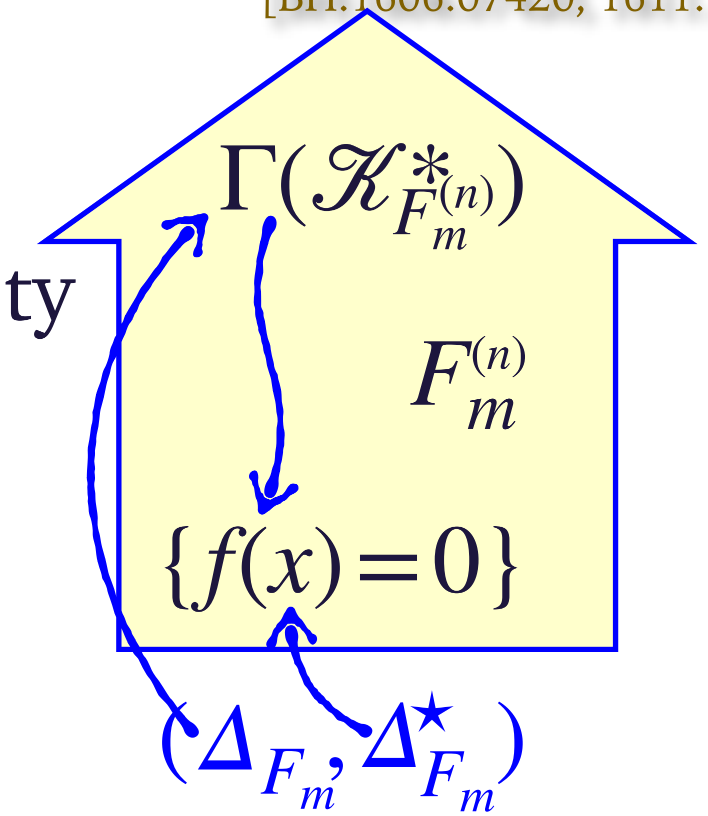
all within the deformation family

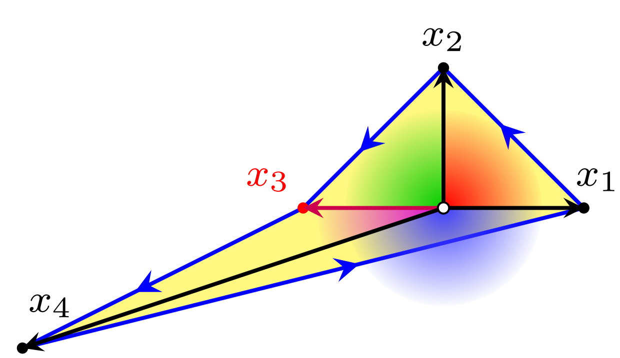
New? Toric Spaces

Do Look Up

[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139]
+more

- Step back for the "big picture"
- Toric (complex algebraic) variety
  - A deformation family of CY hypersurfaces: $F_m^{(n)}[c_1]$
  - In toric-speak (blueprint):





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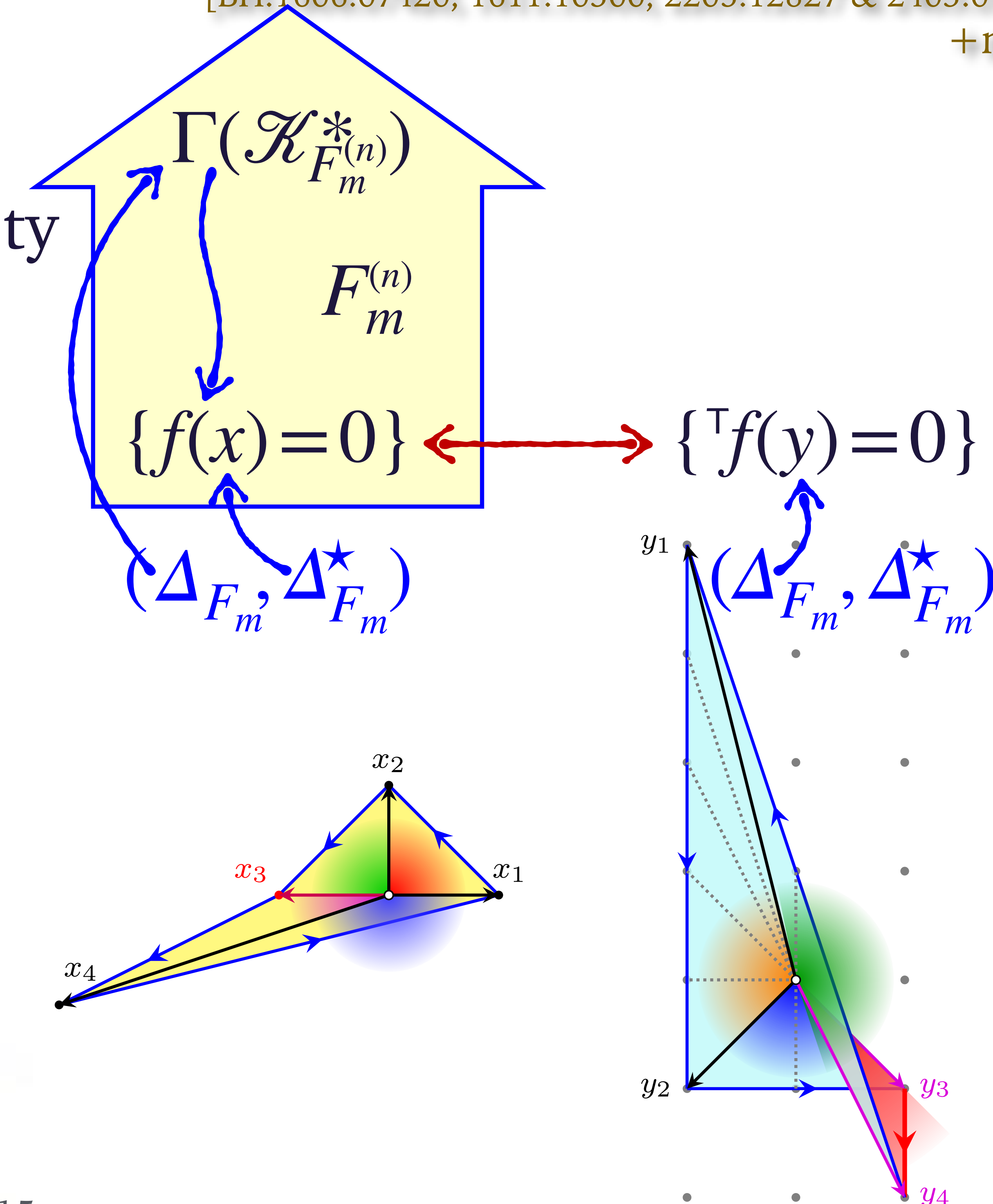

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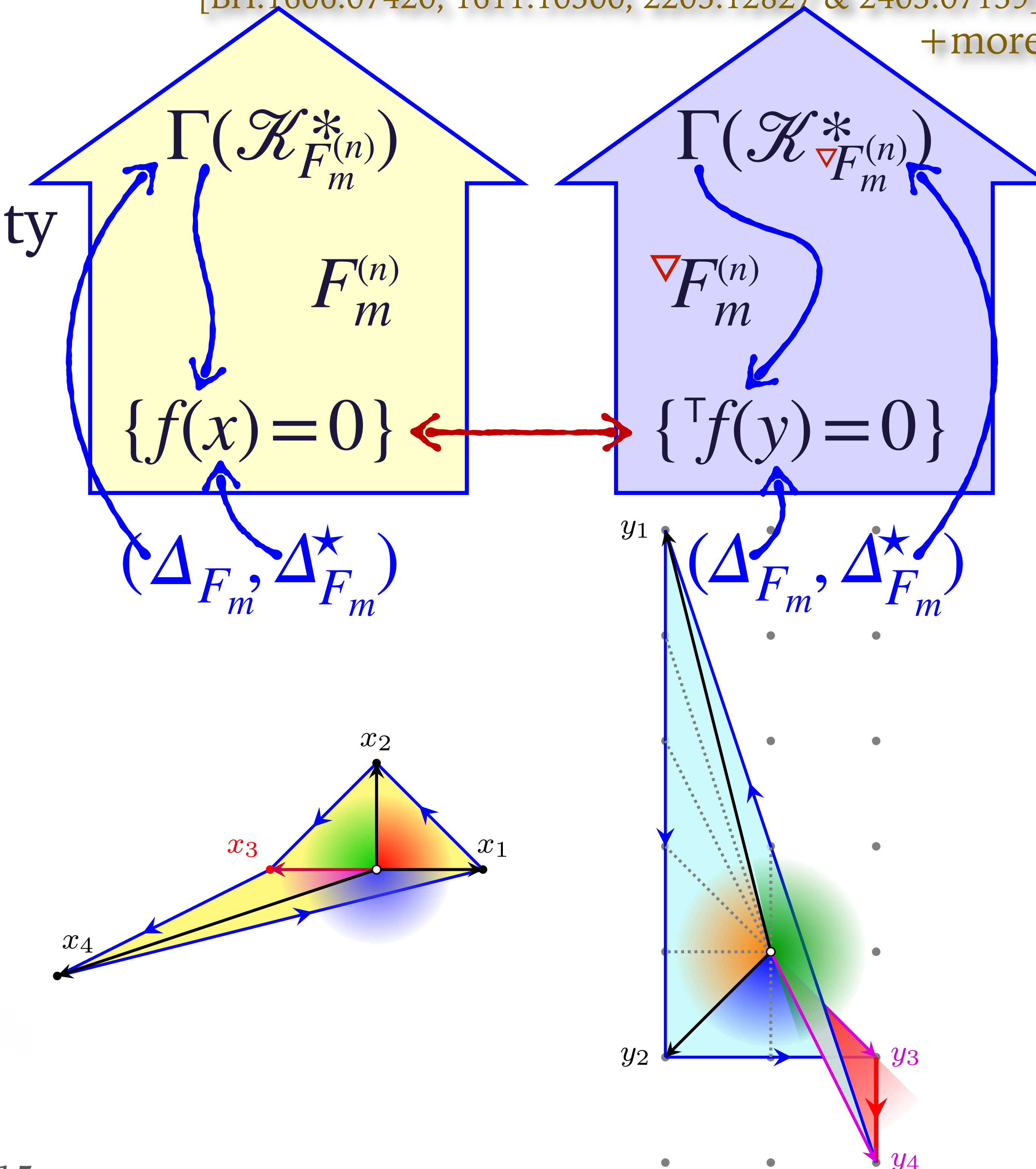

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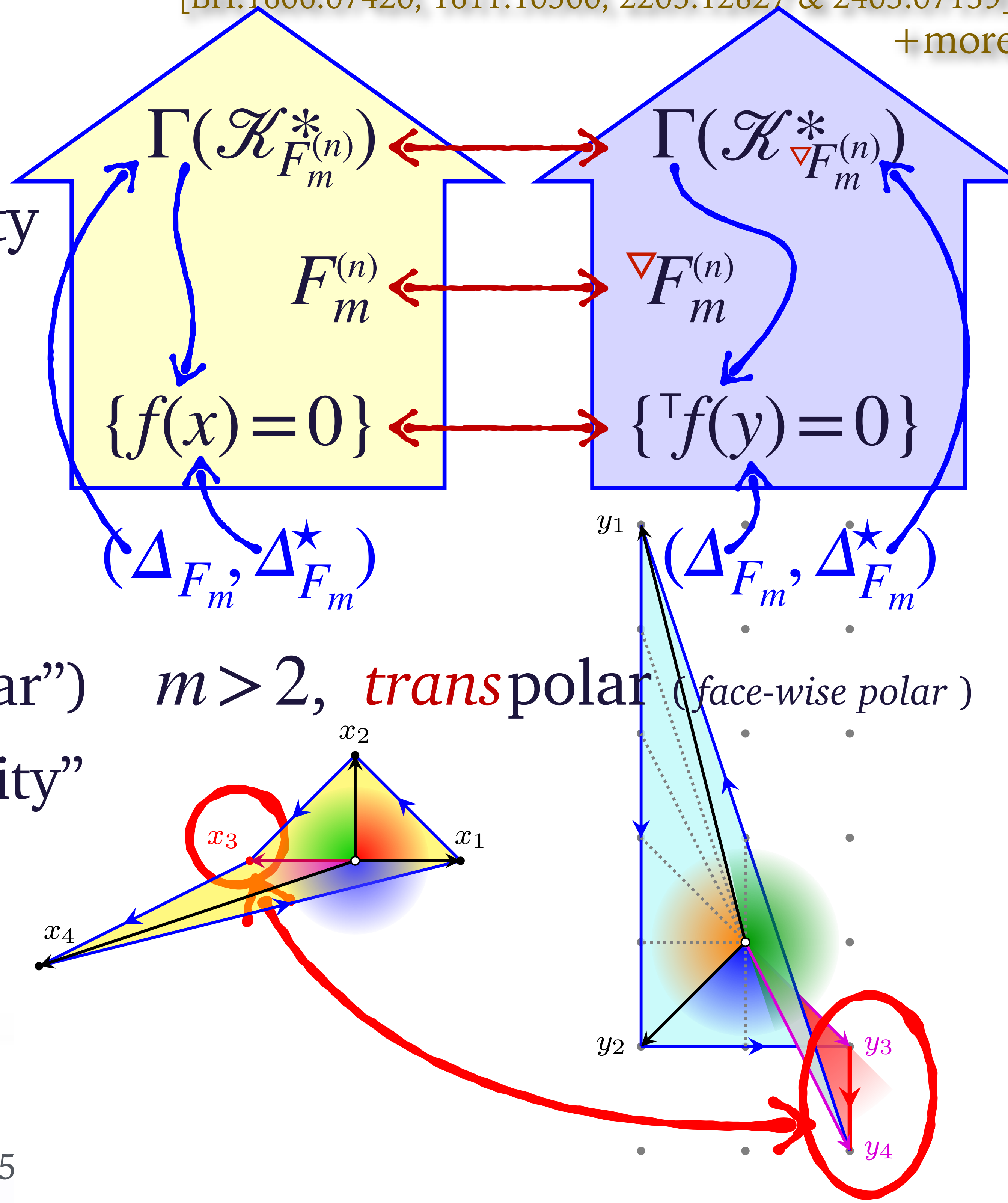
  - The "extension" ↔ "non-convexity" for all $m>2$



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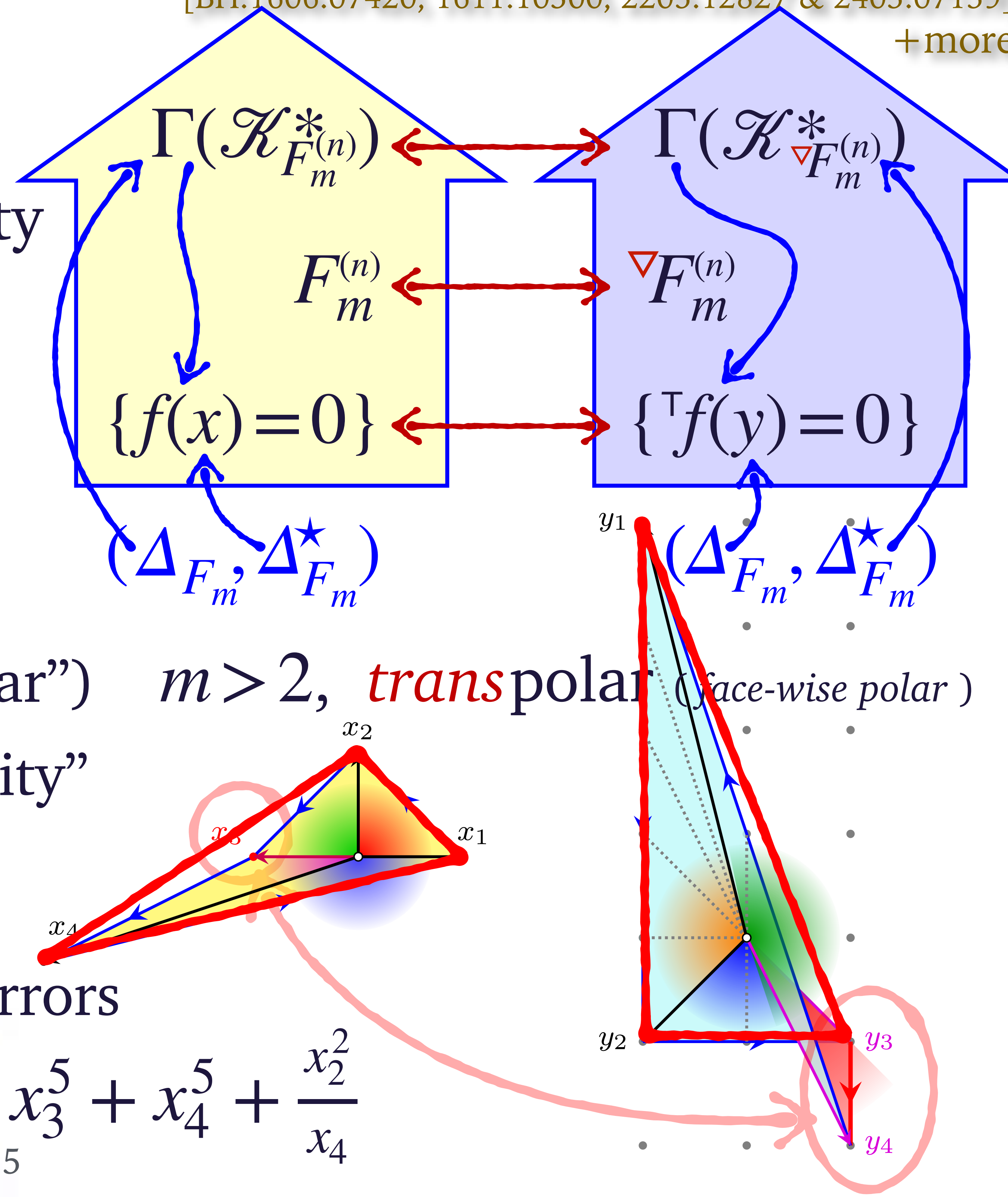

$m>2$, *trans*polar (*face-wise polar*)

$$x_3^5 + x_4^5 + \frac{x_2^2}{x_4}$$

New? Toric Spaces

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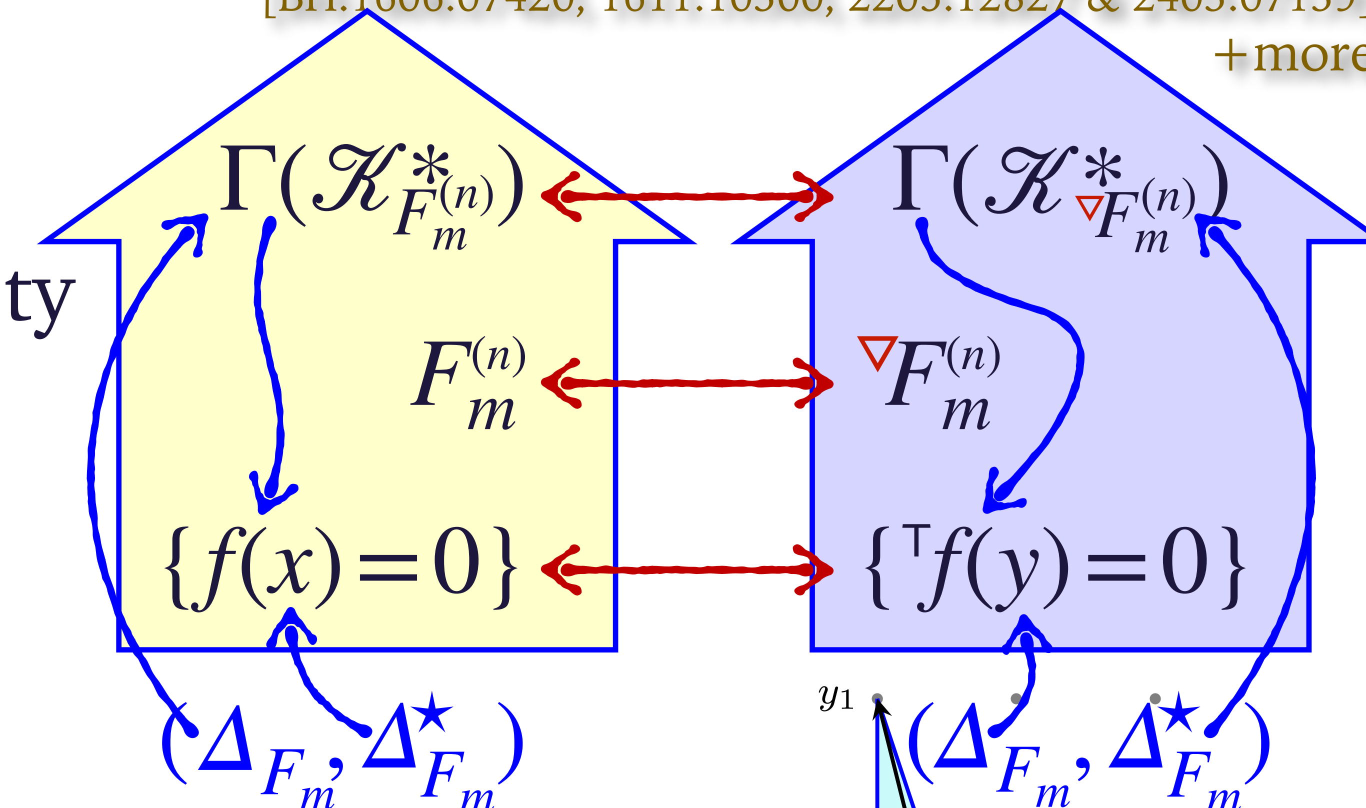

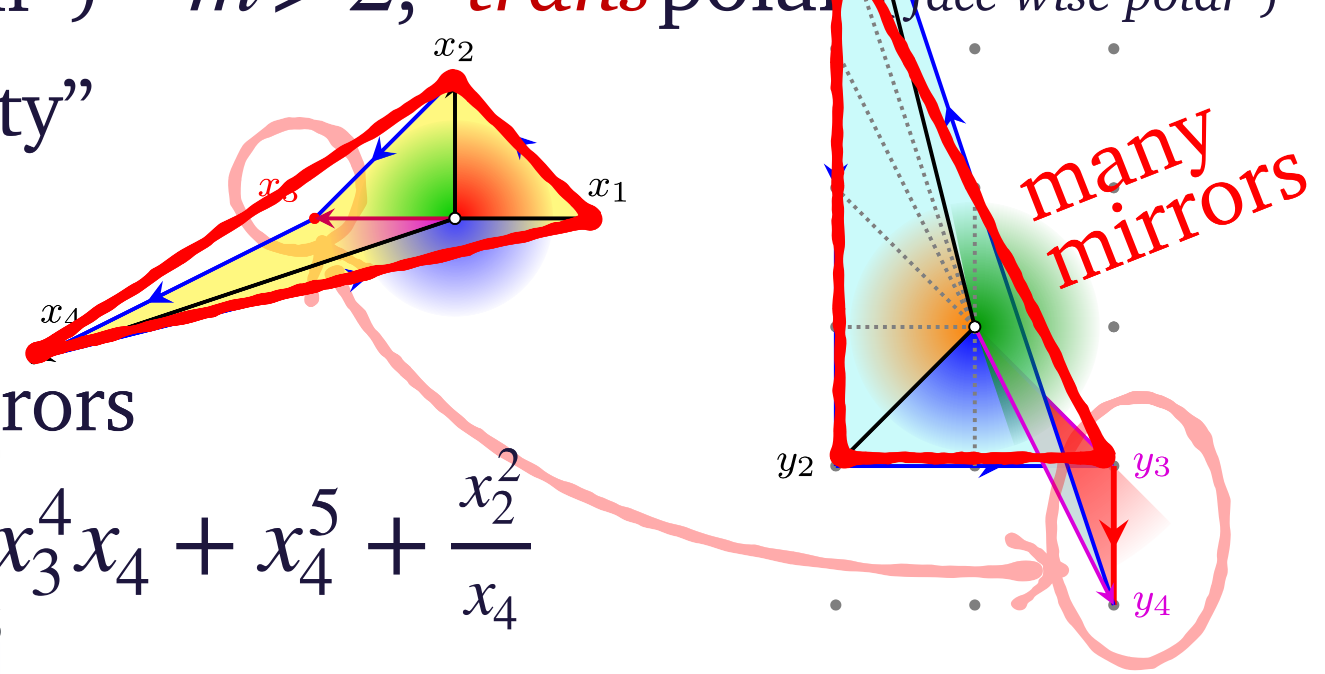

$$x_3^4 x_4 + x_4^5 + \frac{x_2^2}{x_4}$$

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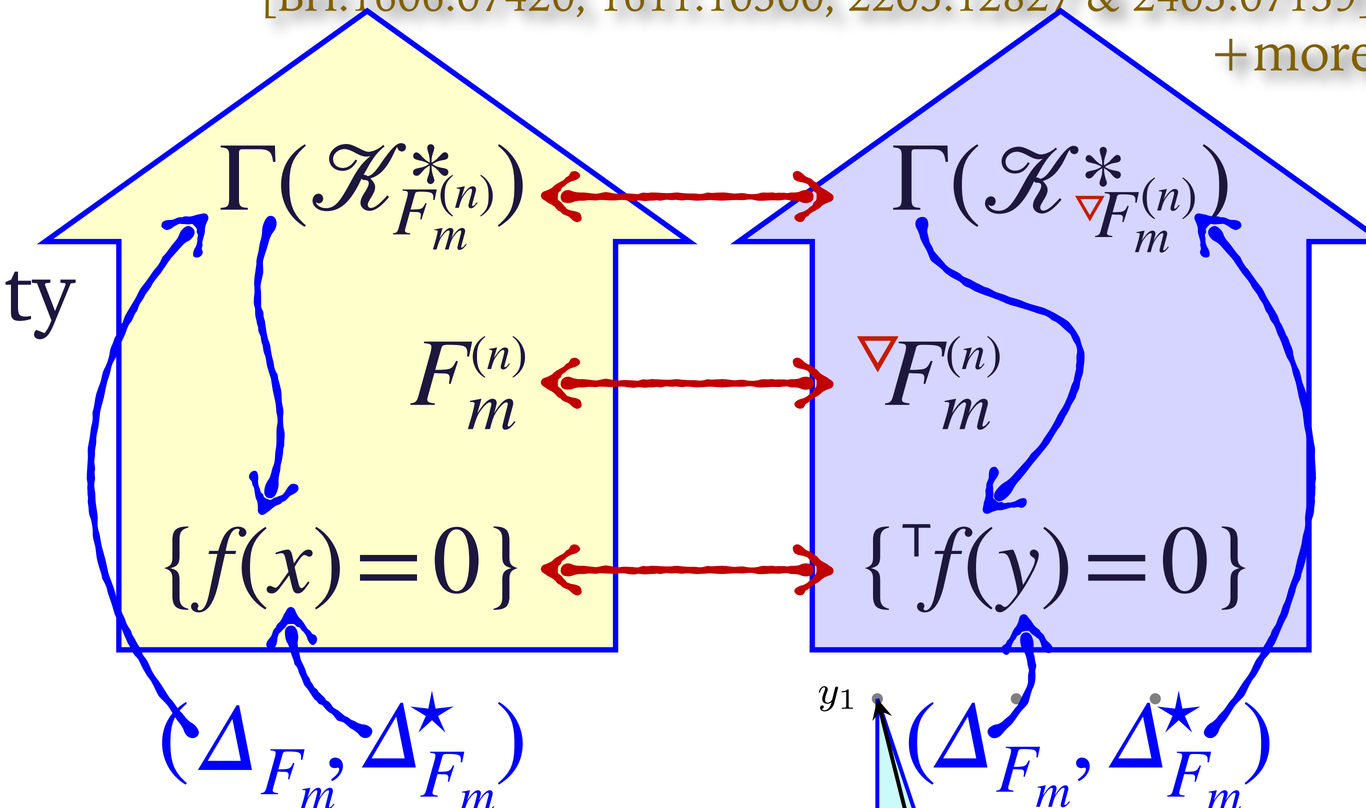

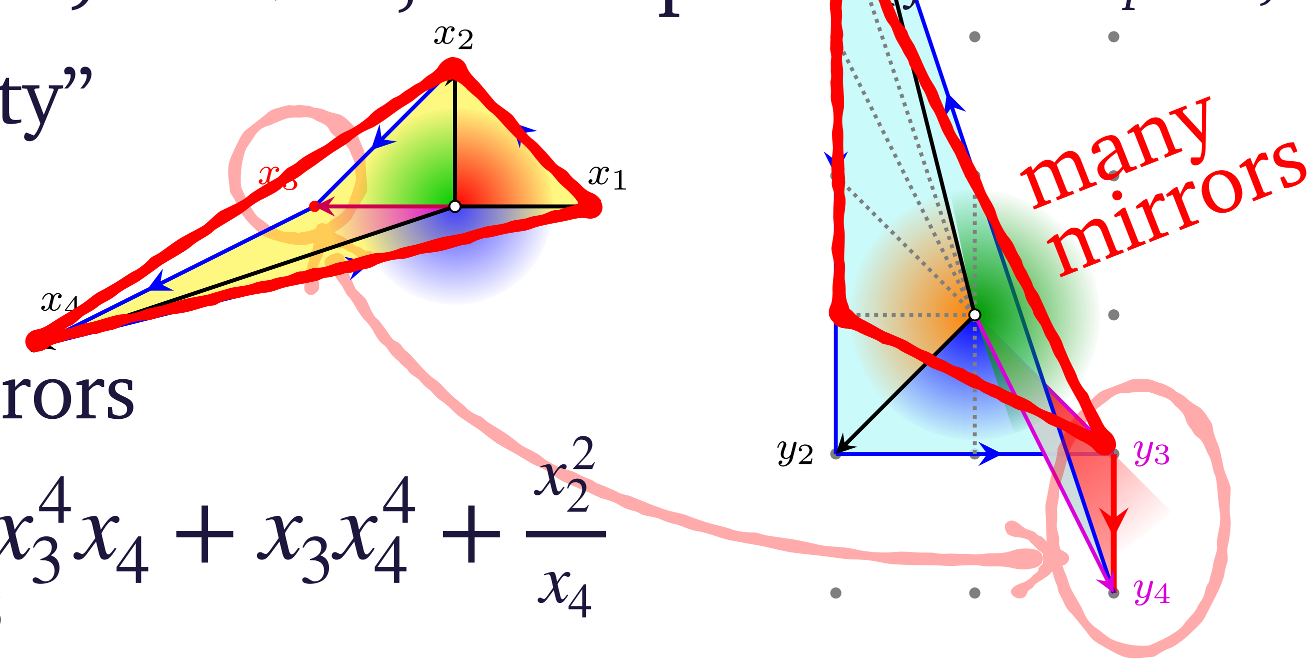

$$x_3^4 x_4 + x_3 x_4^4 + \frac{x_2^2}{x_4}$$

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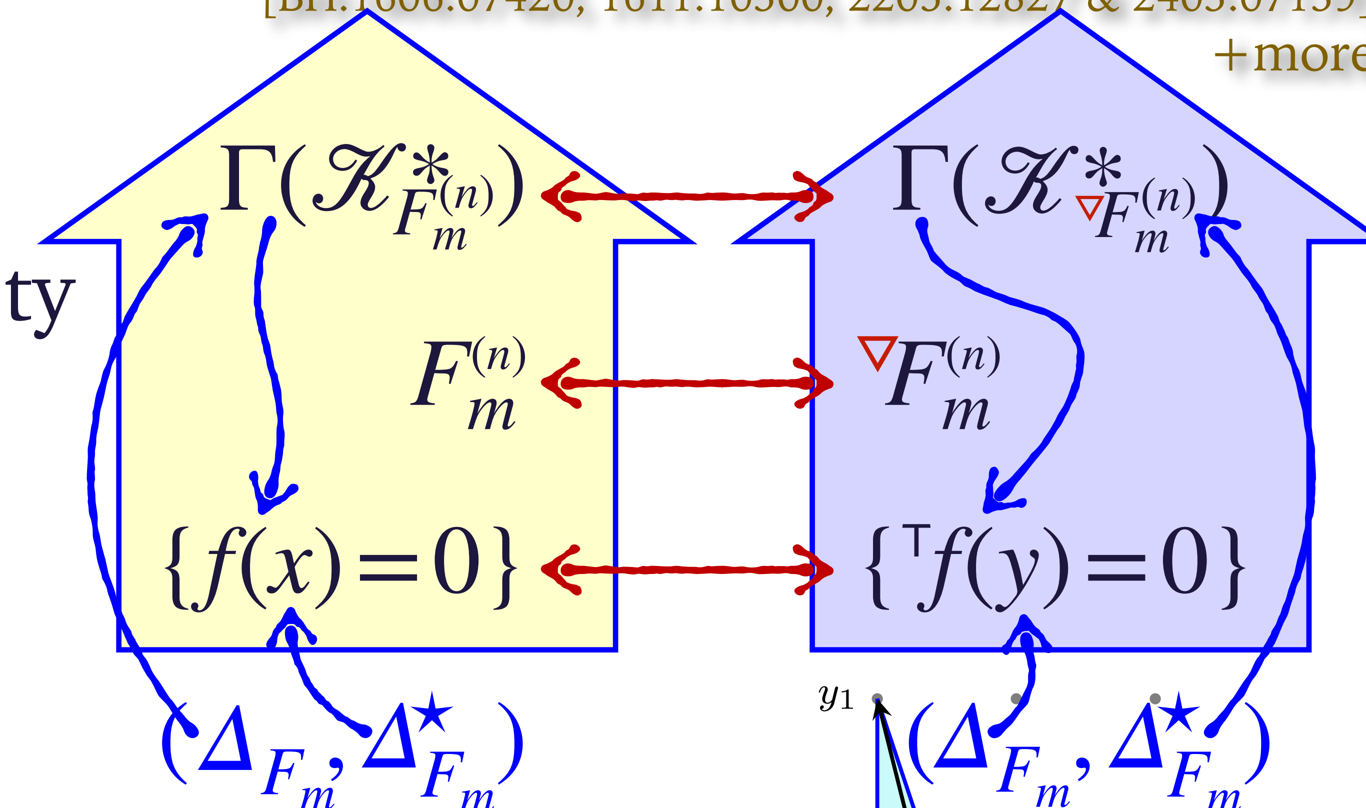

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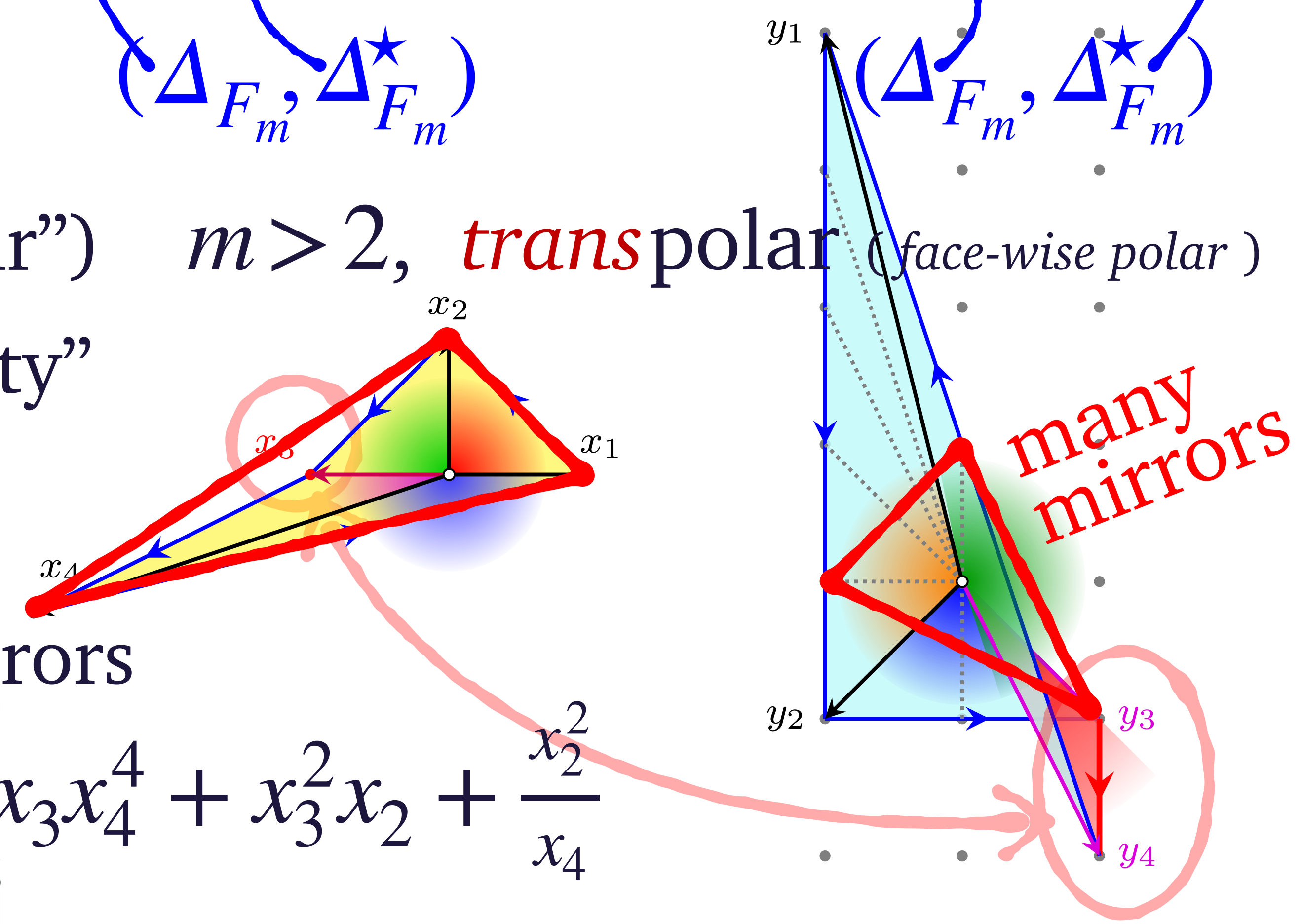

$$x_3 x_4^4 + x_3^2 x_2 + \frac{x_2^2}{x_4}$$

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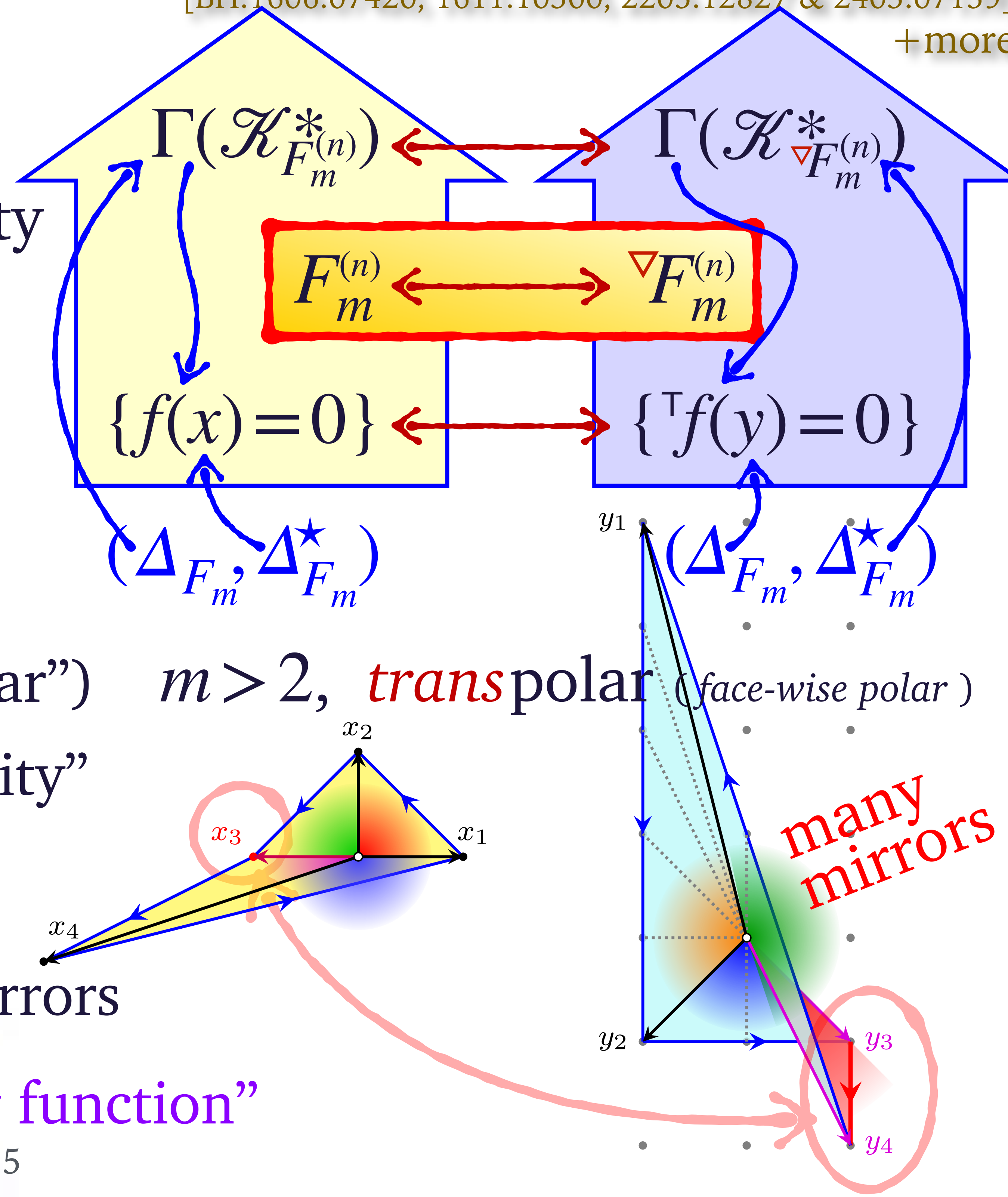
  - Pick simplicial subsets for defining sections → multiple mirrors
- This "big picture" $\stackrel{?}{=}$ "generating function"



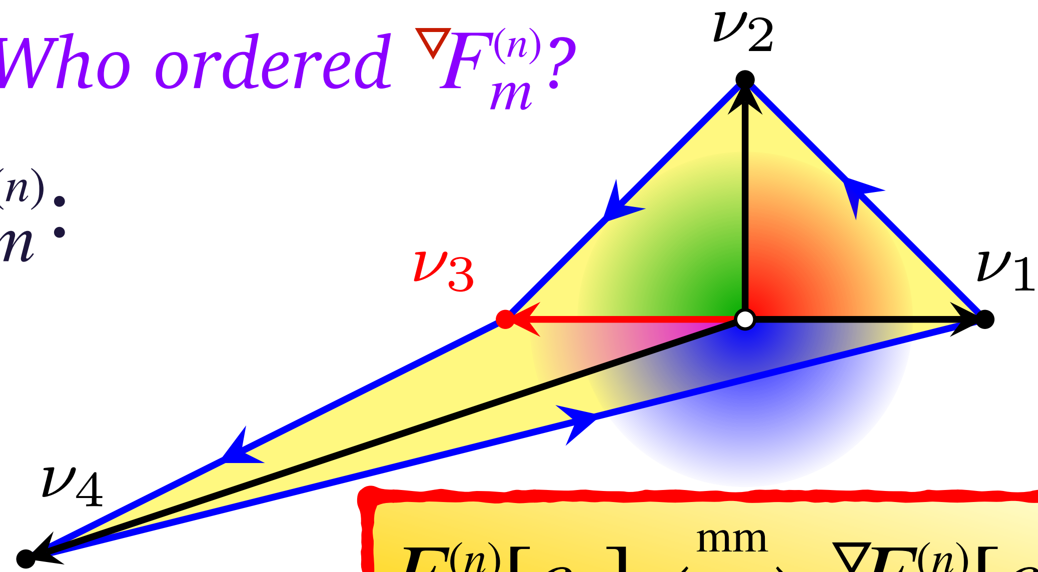
$F_m^{(n)} \longleftrightarrow {}^{\nabla}F_m^{(n)}$

New? Toric Spaces

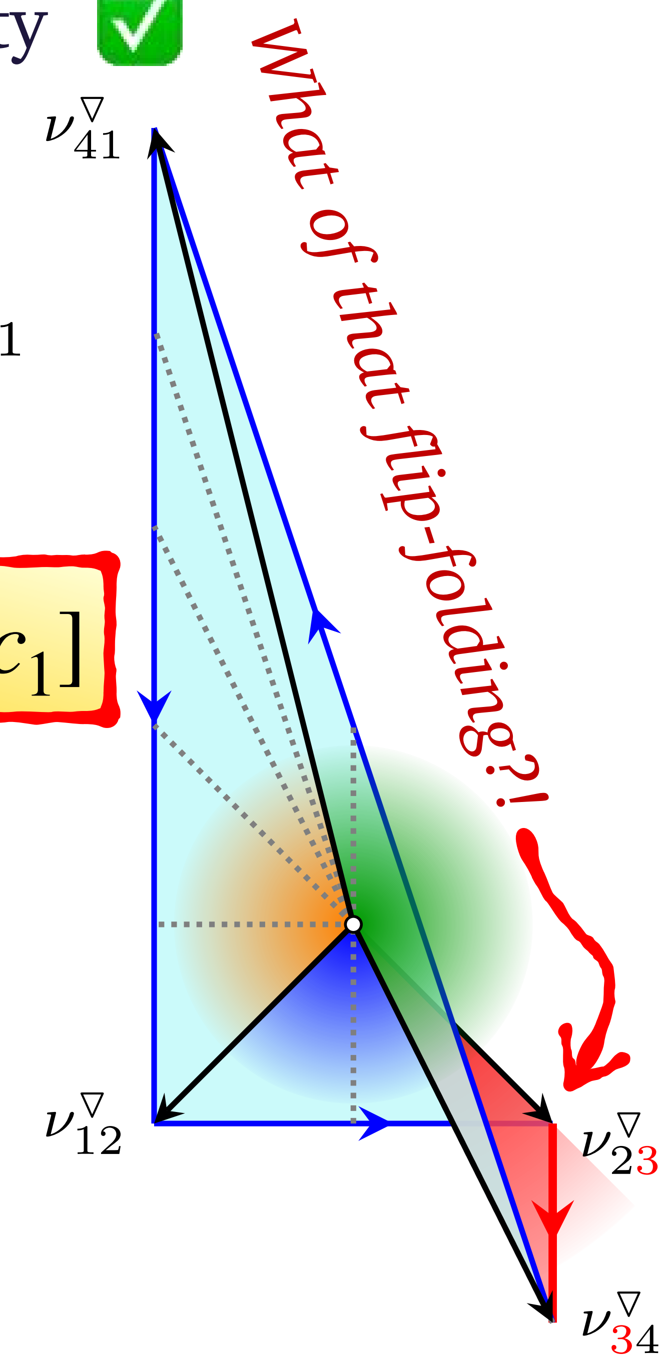
Do Look Up

[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139] +more

- GLSM: $U(1)^n$-gauge symmetry; worldsheet SuSy: $U(1)^n \to (\mathbb{C}^*)^n$
  - Regular monomials ↔ toric (complex algebraic) variety ✅
  - which ${}^{\nabla}F_m^{(n)}$ …isn't. — *Who ordered ${}^{\nabla}F_m^{(n)}$?*
- Just as $\Sigma_{F_m^{(n)}}$ encodes $F_m^{(n)}$:
  - top cone = local chart;
  - codim-1-cone = gluing
- so does its *trans*polar
  - a $2n$-dim *manifold* w/ $U(1)^n$-action
  - the …transpolar of $F_m^{(n)}$, denoted ${}^{\nabla}F_m^{(n)}$
- General multifans (& multitopes) correspond to
  - torus manifolds = ***real $2n$-dim mflds w/ $U(1)^n$-action***
  [Masuda, 1999, 2000; Hattori+Masuda, 2003]



$$F_m^{(n)}[c_1] \overset{\text{mm}}{\longleftrightarrow} {}^{\nabla}F_m^{(n)}[c_1]$$



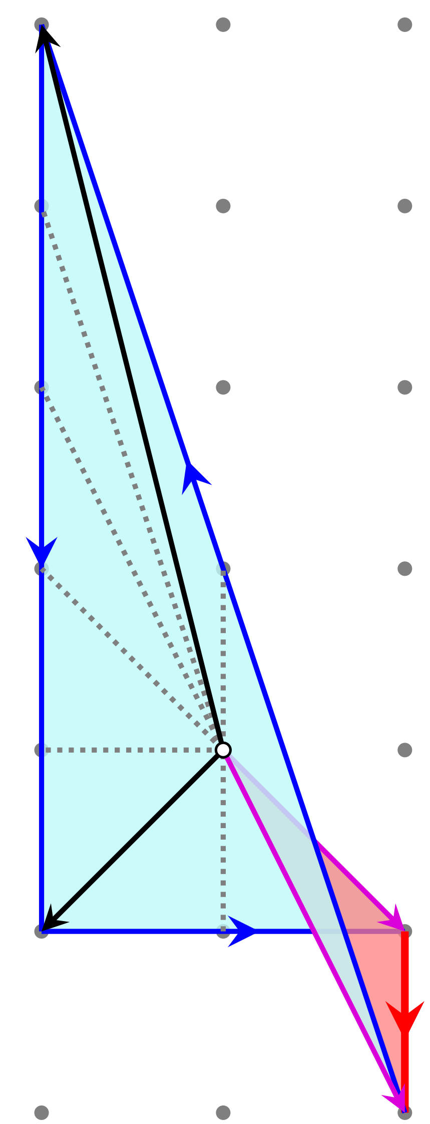
New? Toric Spaces

Do Look Up

Can we now use *all* of it?!

[BH:1606.07420, 1611.10300, 2205.12827 & 2403.07139] +more

- What *is* this "$^{\nabla}\!F_m^{(n)}$" ? (Such that $^{\nabla}\!F_m^{(n)}[c_1] \xleftrightarrow{\text{mm}} F_m^{(n)}[c_1]$?)
  - Fan $\{\sigma_i;\prec\}$ of $\Delta_{F_m^{(n)}}$ ↔ atlas of charts $U_{\sigma_i} \approx \mathbb{C}^n$, $\dim \sigma_i = n$
  - But one chart is oriented reversely…



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  - Fan $\{\sigma_i;\prec\}$ of $\Delta_{F_m^{(n)}} \leftrightarrow$ atlas of charts $U_{\sigma_i} \approx \mathbb{C}^n$, $\dim \sigma_i = n$
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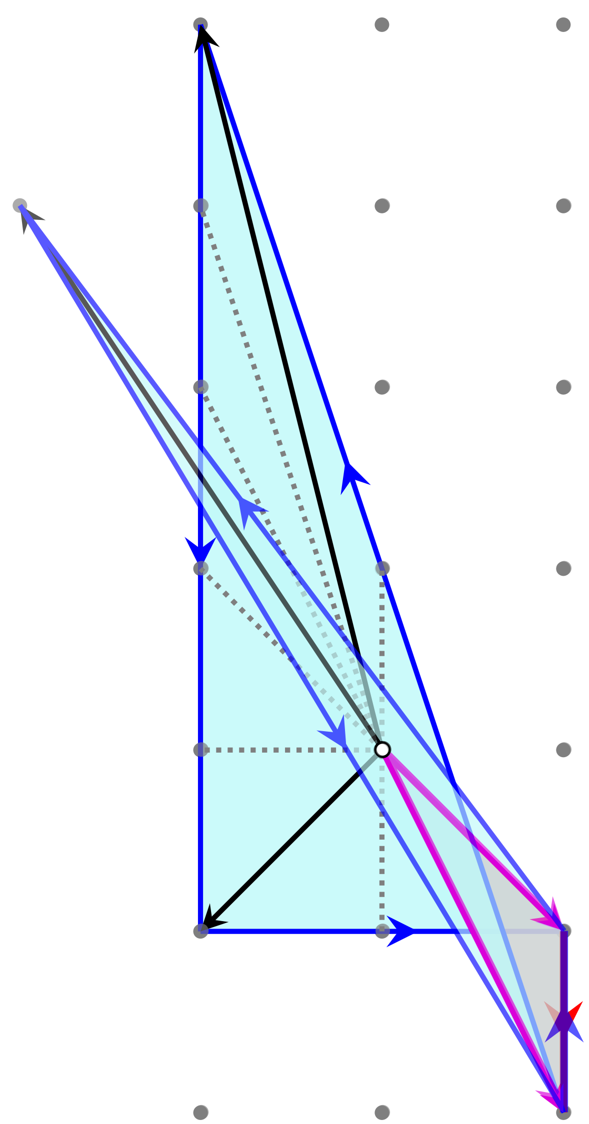
- Every flip-folded cone/facet can be surgically rev.-engineered



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Can we now use <u>all</u> of it?!

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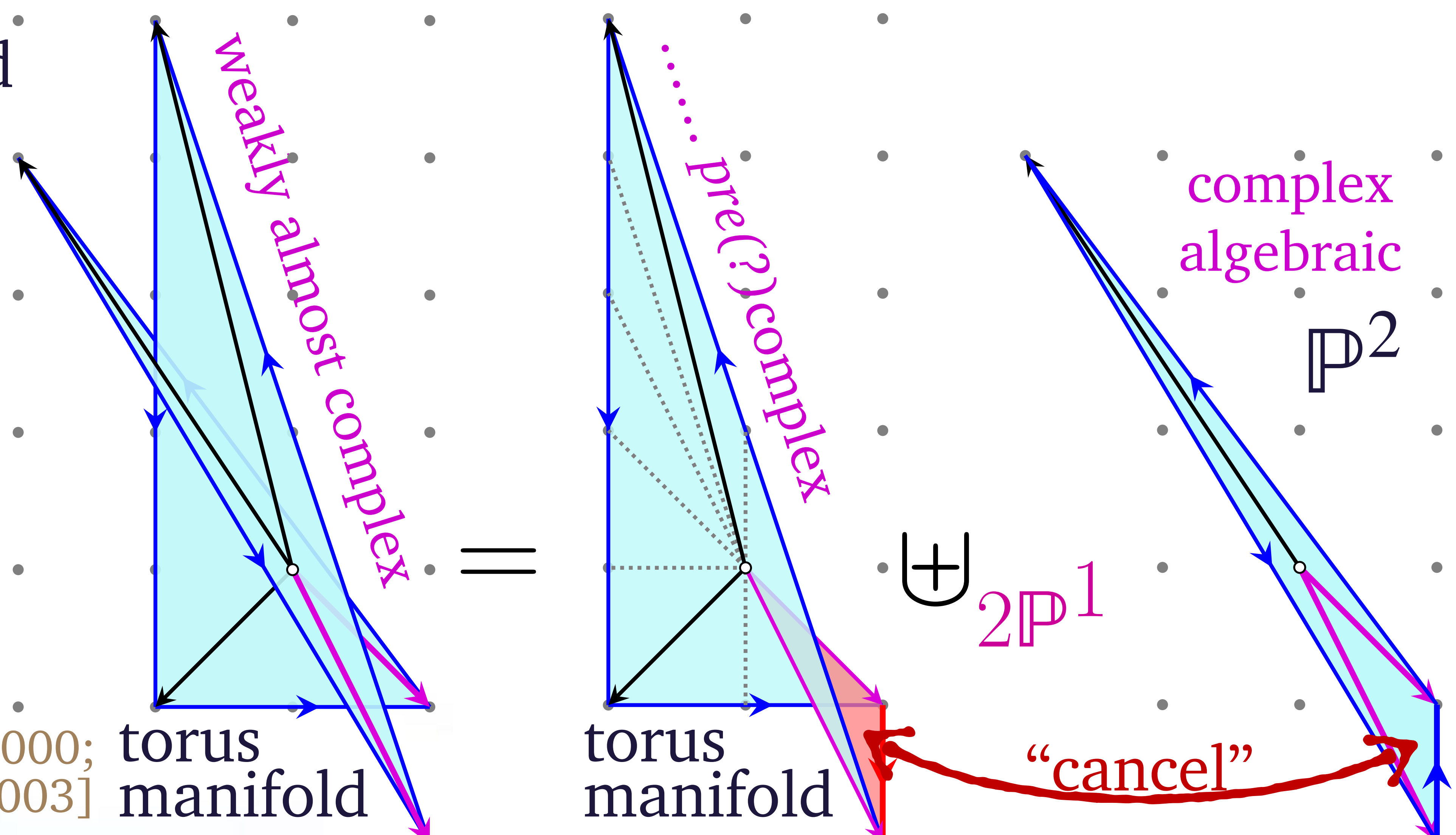
- What <u>*is*</u> this "${}^{\nabla}\!F^{(n)}_m$" ? (Such that ${}^{\nabla}\!F^{(n)}_m[c_1] \xleftrightarrow{\text{mm}} F^{(n)}_m[c_1]$?)
  - Fan $\{\sigma_i; \prec\}$ of $\Delta_{F^{(n)}_m} \leftrightarrow$ atlas of charts $U_{\sigma_i} \approx \mathbb{C}^n$, $\dim \sigma_i = n$
  - But one chart is oriented reversely…
- Every flip-folded cone/facet can be surgically rev.-engineered
- …from regular (cpx. alg.) toric varieties and (non-algebraic) torus manifolds

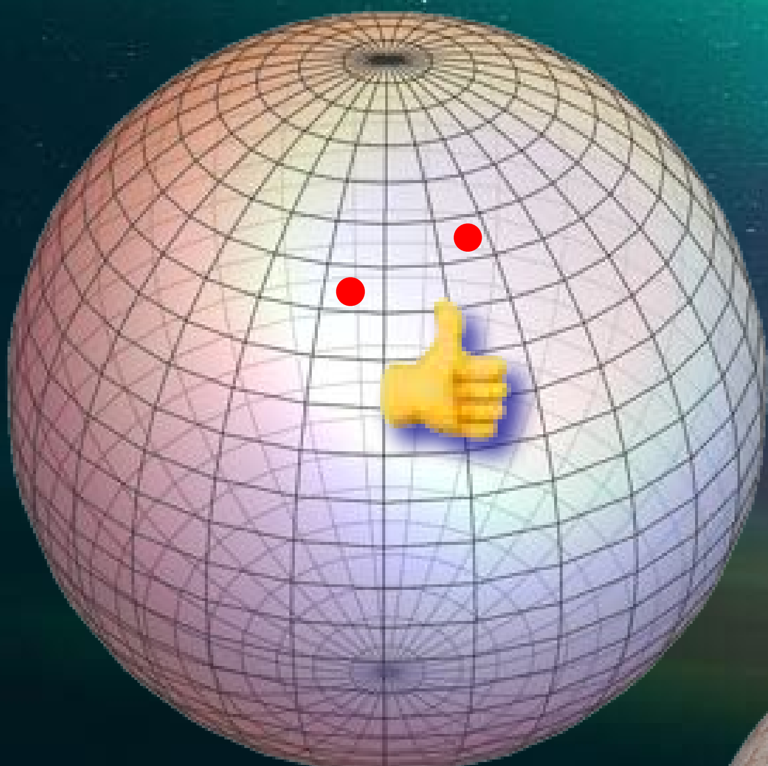
[Masuda, 1999, 2000; Hattori+Masuda, 2003]



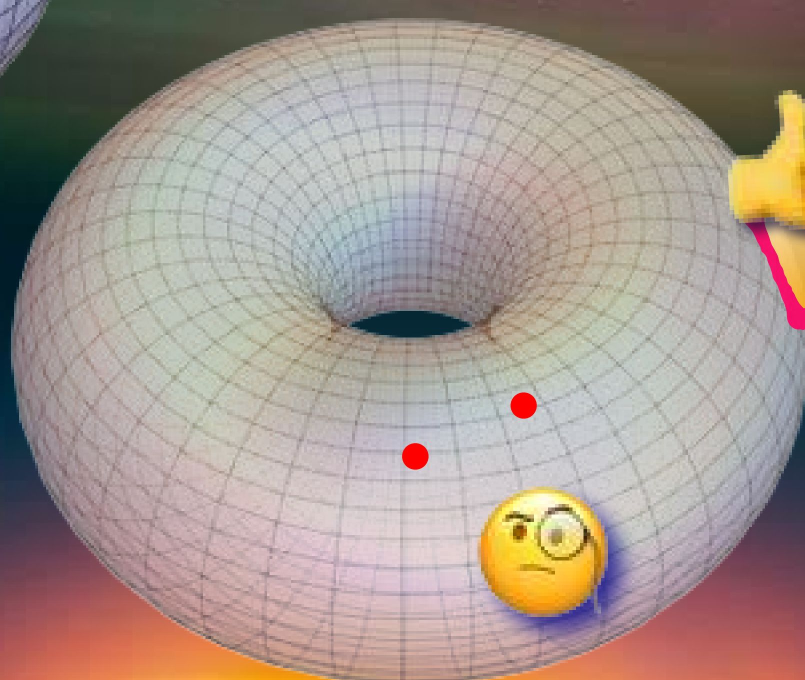
How Hard Can it Be?

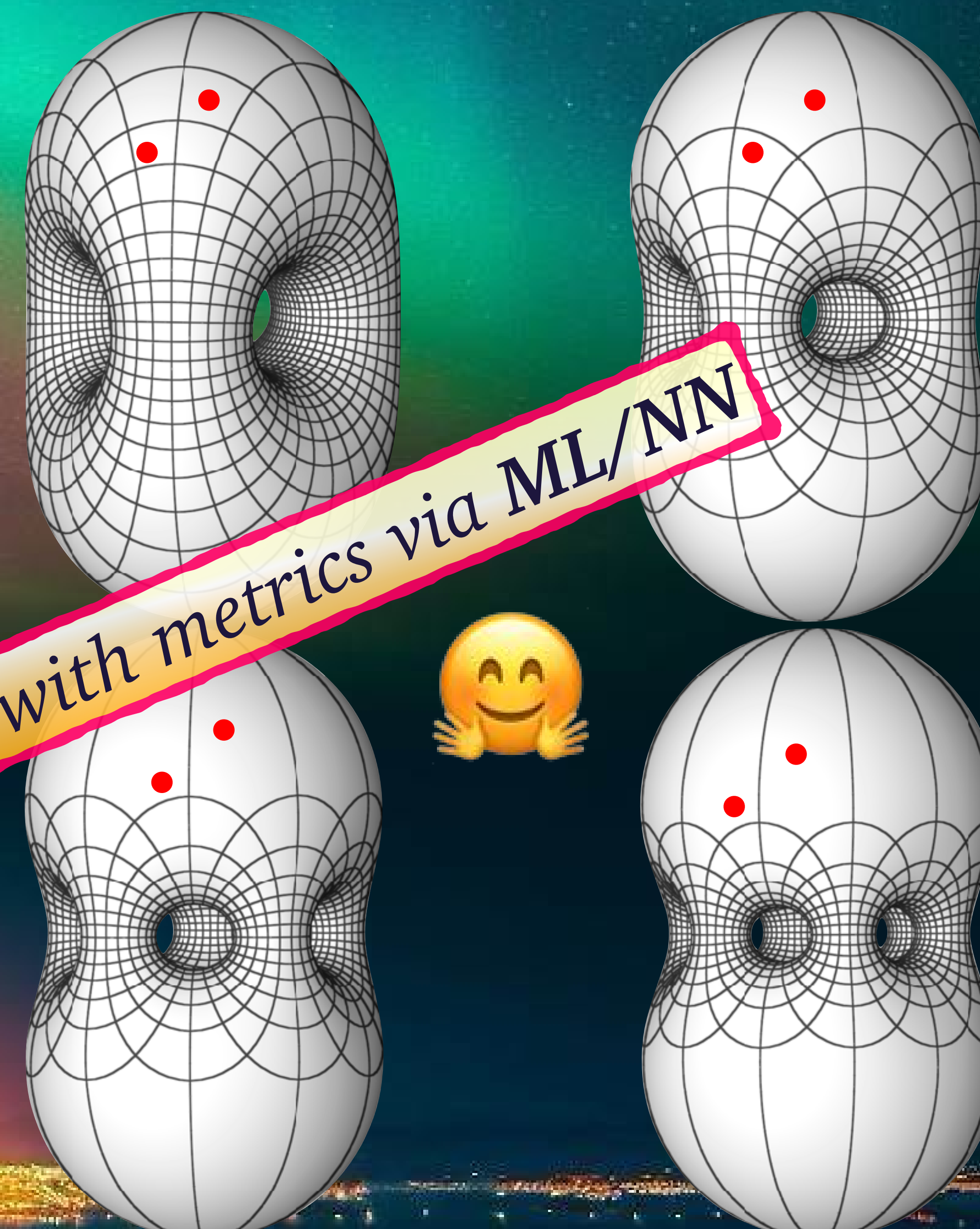
Constructing CY ⊂ Some "Nice" Ambient Space

- Reduce to 0 dimensions: $\mathbb{P}^4[5] \to \mathbb{P}^3[4] \to \mathbb{P}^2[3] \to \mathbb{P}^1[2]$



Just add dimensions, according to taste





...with metrics via ML/NN

Thank You!

Tristan Hübsch

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Department of Physics, Faculty of Natural Sciences, Novi Sad University, Serbia
Department of Mathematics, University of Maryland, College Park, MD

https://tristan.nfshost.com/cv.html