Running coupling in holographic QCD

I.Aref'eva

Steklov Mathematical Institute

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Outlook

- Running coupling in QCD
  - Theory & Experiment

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- Running coupling in QCD at non-zero T
  - Theory & Experiment

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- Running coupling in QCD at non-zero T and chemical potential μ
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  B - magnetic field
  - Theory: **Holographic** QCD

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    NICA=Nuclotron-based Ion Collider fAcility

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  - **Main result**: **Automodel behaviour of the QCD running coupling near 1-st order phase transition**

Running coupling in vacuum QCD. 1/2

The running coupling constant in QCD is a fundamental concept reflecting the energy dependence of the strong interaction.

- **Theory**. The coupling constant α_s varies with the energy scale according asymptotic freedom. This behavior is governed by the RG eq.:

  $$\frac{d\alpha_s(M^2)}{d\ln M^2} = -\beta(\alpha_s(M^2)),\ M - \text{renormaliz scale},\ \beta(\alpha_s) - \beta\text{function of QCD}$$

  M is the renormalization scale and $\beta(\alpha_s)$ is the beta function of QCD.

  At 1-loop order $\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2, \quad \beta_0 = 11 - \frac{2}{3}n_f$, n_f is # of flavors.
  The running coupling constant: $\alpha_s(Q^2) = \frac{2\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}$, Λ_{QCD} is the QCD scale parameter ∼200-300 MeV.

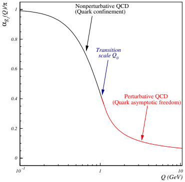
Running coupling in vacuum QCD. 2/2

- **Experimental Results.** The running of $\alpha_s(Q^2)$ has been confirmed experimentally over a wide range of energy scales. Measurements of α_s at various scales Q came from:

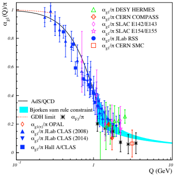
- Deep inelastic scattering (DIS)
- e^+e^- annihilation into hadrons
- The hadronic decays of the Z boson
- Lattice QCD calculations

- PDG provides a world average of the strong coupling constant at the Z boson mass scale, $M_Z = 91.1876 GeV$:

$$\alpha_s(M_Z^2) = 0.1181 \pm 0.0011.$$



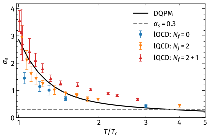
A)



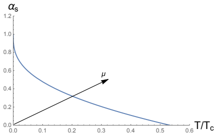
B)

A) Unified coupling matching of nonperturbative and perturbative QCD regimes.
B) Experimental data and sum rule constraints for the effective charge α_{g_1}, from: S.Brodsky at all, 2403.16126 and earlier

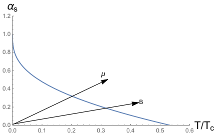
Running coupling in QCD.
What is known/expected for $T, \mu, B \neq 0$



A)



B)



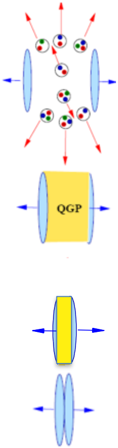
C)

A) Running coupling constant α as a function of T/T_c for $\mu = 0$, from: 2308.03105

B) It is expected that $\alpha(T, \mu) = f(T^2 + c\mu^2)$.

C) $\alpha(T, \mu, B) = ?$

Evolution during heavy ion collision



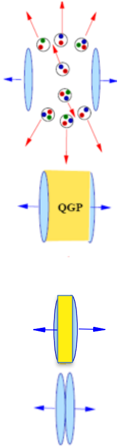
Адронизация

Время жизни:
10fm/c

Время термализации:
$1fm/c = 3.3 \cdot 10^{-24}s$

- QGP is a state of matter of free quarks, antiquarks and gluons at high temperature. QGP was discovered at RHIC in 2005.
- QGP behaves (RHIC, LHC) like a strongly interacting fluid (collective effects)

Evolution during heavy ion collision



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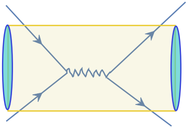


QGP - strongly interacting liquid

- **Two questions:**

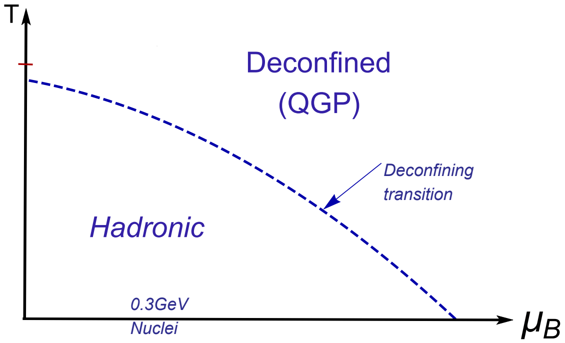
  1. How was it formed?
  2. What properties does it have?

- **The main property is the structure of the phase diagram**
- **One of the goals of NICA - study the phase structure of QCD**

- **Main message from this talk**: scattering amplitudes in for processes in area of HIC can feel location of 1st order phase transition in $(\mathbf{T}, \boldsymbol{\mu}, \mathbf{B})$-space

QCD Phase Diagram: Early Conjecture

Cabibbo and Parisi, 1975

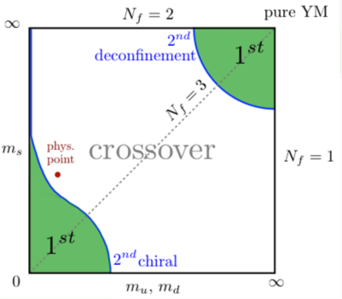


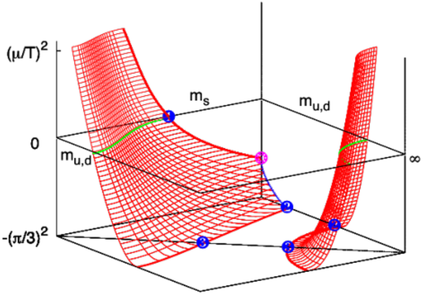
- μ a measure of the imbalance between quarks and antiquarks in the system

QCD Phase Diagram: Lattice

Phase diagram
on quark mass

Main problem with $\mu \neq 0$
Imaginary chemical potential method



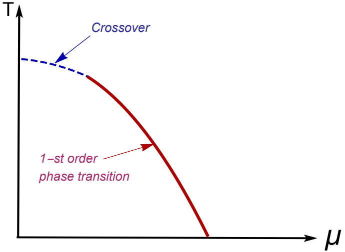


Columbia plot
Brown et al., PRL (1990)

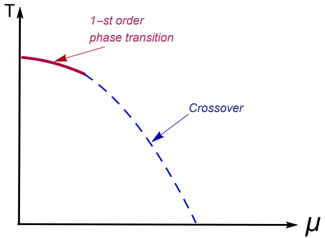
Philipsen, Pinke, PRD (2016)

“Heavy” and “light” quarks from Columbia plot

Light quarks



Heavy quarks



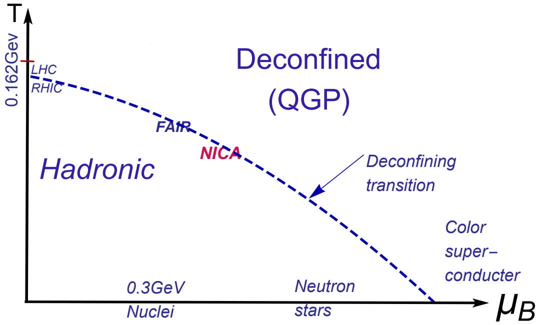
QCD Phase Diagram: Experiments

- LHC, RHIC (2005);
- FAIR (Facility for Antiproton and Ion Research),

  NICA (Nuclotron-based Ion Collider fAcility)

Main goals

- search for signs of the phase transition between hadronic matter and QGP;
- search for new phases of baryonic matter



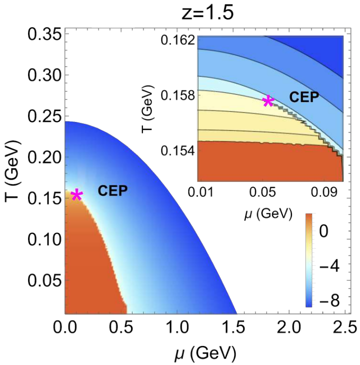
QCD Phase Diagram: Experiments

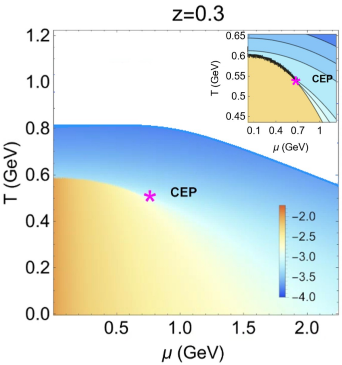
- Main experimental method to find 1-st order phase transition:
  Beam Energy Scan Method

- We propose **a new method** related with study of scattering amplitudes of particles created in colliding heavy ions beams.

- It is related with **special behaviour of scattering amplitudes near 1-st order phase transition**

“Light” and “Heavy” quarks phase diagrams from scattering amplitudes. B=0

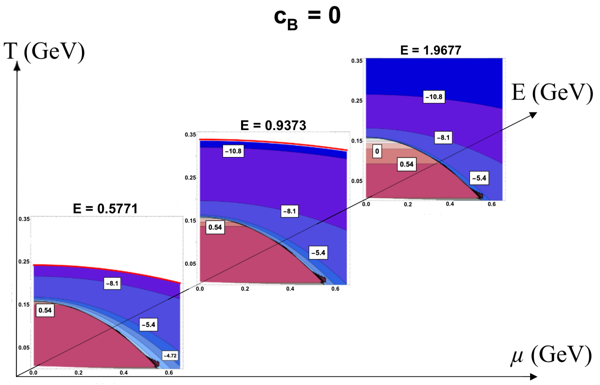




Density plots of $\log \alpha(z;\mu,T)$ for light quarks and heavy quarks

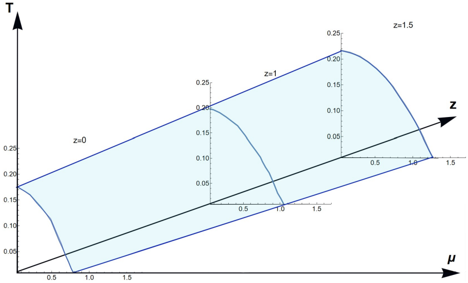
This is predicted by Holographic QCD

"Light"quarks phase diagrams



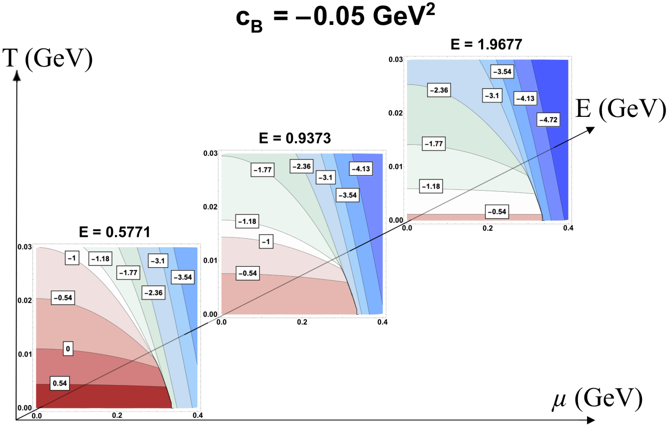
Density plots of $\log \alpha(E;\mu,T)$ at different energy scales.
All values of E on the top of each panel show fixed value of energy E-coordinate.

Automodel behaviour of Running coupling



$$\alpha(T,\mu) = f_{up}(T^2 + c\mu^2)$$

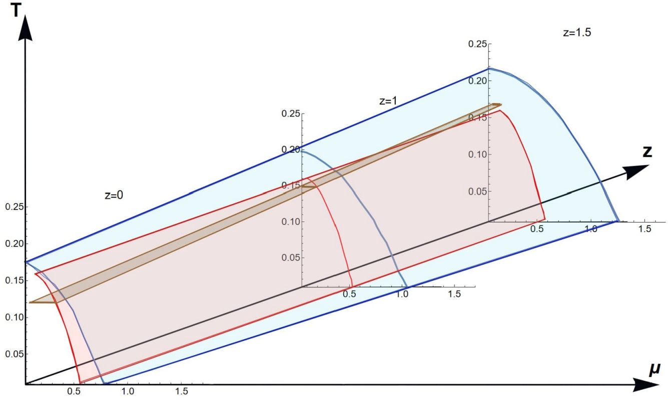
"Light"quarks phase diagrams. $B \neq 0$



Density plots of $\log \alpha(E;\mu,T)$ at different energy scales for $c_B = -0.05\ \text{GeV}^2$.

I.A. A.Hajilou, A.Nikolaev, P.Slepov, 2407.11924

Automodel behaviour of Running coupling



$$\alpha(T, \mu) = f_{below}(T^2 + c\mu^2)$$

Holographic QCD

- Perturbation methods are not applicable to describe QCD phase diagram
- Lattice methods do not work, because of problems with the chemical potential.
- Holographic QCD - phenomenological model(s)

- One of goals of Holographic QCD – describe QCD phase diagram

- **Requirements:**
  - reproduce the QCD results from perturbation theory at short distances
  - reproduce Lattice QCD results at large distances (~ 1 fm) and **small** μ_B

Holographic method - phenomenological approach

Motivated by AdS/CFT duality Maldacena,1998

- Temperature in QCD $\Longleftrightarrow$ black hole temperature in (deform.)AdS
- Thermalization in QCD $\Longleftrightarrow$ formation of black hole in (deform.)AdS5
- Thermalization models (black hole formation models): colliding shock waves; the area of the trapped surface determines the multiplicity

Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S=\int d^5x\,\sqrt{-g}\left[R-\frac{f_1(\phi)}{4}\,F_{(1)}^2-\frac{f_B(\phi)}{4}\,F_{(B)}^2-\frac{1}{2}\,\partial_M\phi\partial^M\phi-V(\phi)\right]$$

$$ds^2=\frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-g(z)\,dt^2+dx^2+dy_1^2+e^{c_Bz^2}dy_2^2+\frac{dz^2}{g(z)}\right]$$

$A_{(1),m}=A_t(z)\delta_m^0,\ A_t(0)=\mu, \qquad F_{(B)}=dx\wedge dy^1$

Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$$\mathfrak{b}(z)=e^{2\mathcal{A}(z)}$$ $\Leftrightarrow$ quarks mass **"Bottom-up approach"**

Heavy quarks (b, t):

$\mathcal{A}(z)=-cz^2/4$ *Andreev, Zakharov'06*

$\mathcal{A}(z)=-cz^2/4+pz^4$ *IA, Hajilou, Rannu, Slepov, 2305.06345*

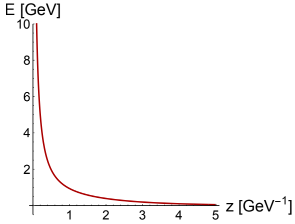
Light quarks (d, u)

$\mathcal{A}(z)=-a\ln(bz^2+1)$ *Li, Yang, Yuan'17*

φ - dilaton, $\alpha(z)=e^{\varphi(z)}$ - **running** coupling in HQCD

Running coupling in QCD vs running coupling in HQCD

- $\alpha(z) = e^{\varphi(z)}$ running coupling in HQCD, here φ - dilaton,
  E. Kiritsis et al 1401.0888, 1805.01769
- $\alpha = \alpha(E)$ - running coupling in QCD
- The energy scale as a function of the bulk coordinate z, corresponding to the warp factor for light quark model



Energy scale $E = \frac{\sqrt{\mathfrak{b}(z)}}{z}$

Holographic Equation of Motions

$$
\begin{aligned}
\varphi'' + \varphi' \left(\frac{g'}{g} + \frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{3}{z} + c_B z \right) + \left(\frac{z}{L} \right)^2 \frac{\partial f_1}{\partial \varphi} \frac{(A_t')^2}{2\mathfrak{b} g} - \left(\frac{z}{L} \right)^2 \frac{\partial f_B}{\partial \varphi} \frac{q_B^2}{2\mathfrak{b} g} &= 0\,, \\
A_t'' + A_t' \left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f_1'}{f_1} - \frac{1}{z} + c_B z \right) &= 0\,, \\
g'' + g' \left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{3}{z} + c_B z \right) - \left(\frac{z}{L} \right)^2 \frac{f_1 (A_t')^2}{\mathfrak{b}} - \left(\frac{z}{L} \right)^2 \frac{q_B^2\, f_B}{\mathfrak{b}} &= 0\,, \\
\mathfrak{b}'' - \frac{3(\mathfrak{b}')^2}{2\mathfrak{b}} + \frac{2\mathfrak{b}'}{z} - \frac{4\mathfrak{b}}{3z^2} \left(- \frac{c_B z^2}{2} - \frac{c_B^2 z^4}{2} \right) + \frac{\mathfrak{b}\, (\varphi')^2}{3} &= 0\,, \\
c_B z^2 \left(2g' + 3g \right) \left(\frac{\mathfrak{b}'}{\mathfrak{b}} - \frac{4}{3z} + \frac{2c_B z}{3} \right) - \left(\frac{z}{L} \right)^3 \frac{L\, q_B^2 f_B}{\mathfrak{b}} &= 0\,, \\
\frac{\mathfrak{b}''}{\mathfrak{b}} + \frac{(\mathfrak{b}')^2}{2\mathfrak{b}^2} + \frac{3\mathfrak{b}'}{\mathfrak{b}} \left(\frac{g'}{2g} - \frac{2}{z} + \frac{2c_B z}{3} \right) - \frac{g'}{3zg} \left(9 - 3c_B z^2 \right) - \frac{2c_B}{3} \left(5 - c_B z^2 \right) & \\
+ \frac{8}{z^2} + \frac{g''}{3g} + \frac{2}{3} \left(\frac{L}{z} \right)^2 \frac{\mathfrak{b} V}{g} &= 0\,,
\end{aligned}
$$

We choose (LQ): $f_1 = e^{-cz^2 - \mathcal{A}(z)} = (1 + bz^2)^a \, e^{-cz^2}$,

B.C.: $A_t(0) = \mu$, $A_t(z_h) = 0$; $g(0) = 1$, $\quad g(z_h) = 0; \varphi(z_0) = 0$

Holographic Running coupling

$\alpha(z) = e^{\varphi(z)}$ I.A. A.Hajilou, P.Slepov, M.Usova, 2402.14512

Light Quark Model

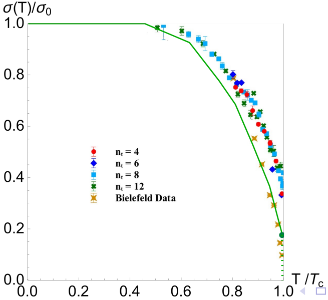
$\varphi(z)$ - dilaton field $\varphi(z)$ is defined up to a constant: $\varphi(z)\Big|_{z=z_0} = 0.$

There are 3 choices: a) $z_0 = 0$ b) $z_0 = f(z_h)$ c) $z_0 = z_h$

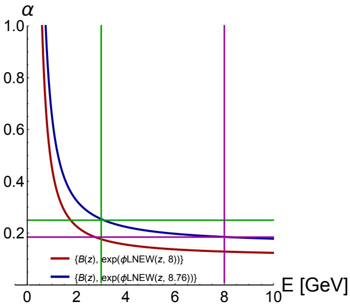
$$z_0 = 10\exp[-z_h/4] + 0.1$$

IA, K.Rannu, P.Slepov, JHEP'21

With this boundary condition the temperature dependence of σ_s fits the known lattice data Cordaso, Bicudo 1111.1317

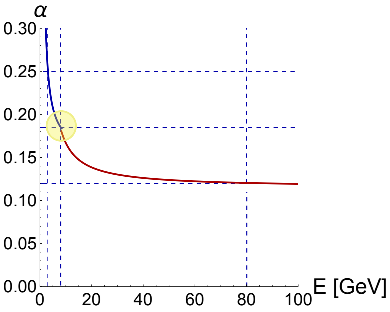


Running coupling in HQCD. $T = 0$



The holographic running coupling for light quarks as a function of the energy scale at $T = 0$. $z_h = 8.76$ (blue), $z_h = 8$ (darker red)

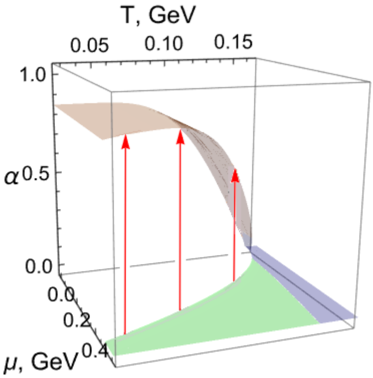
The energy scale $E = \frac{\sqrt{\mathfrak{b}(z)}}{z}$



The holographic running coupling approximated by the heavy quark solution for large E (red line) and by the light quark solution for small E (blue line) as a function of the energy scale at $T = 0$. The yellow disk shows the region where, for the hybrid model, we expect corrections to the approximation by solutions for light and heavy quarks at high and low energies, respectively

Holographic Running coupling for $T \neq 0, \mu \neq 0$

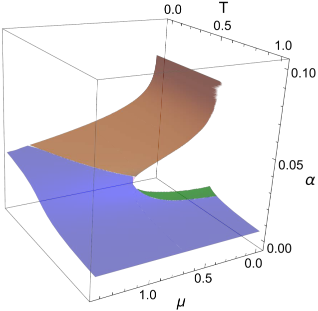
I.A. A.Hajilou, P.Slepov, M.Usova, 2402.14512



Here $z = 0$

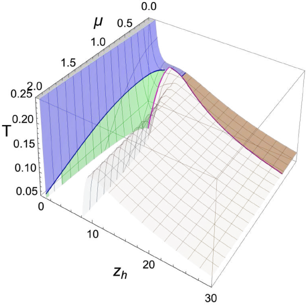
Light quark model

Running coupling for Heavy Quark Model

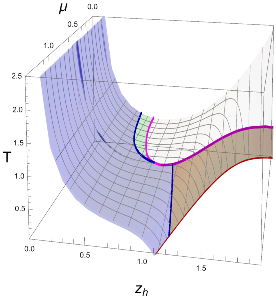


The 3D-plot for coupling constant $\alpha = \alpha_{_{HQ}}(z;\mu,T)$ for heavy quarks at $z=0$. Hadronic, QGP and quarkyonic phases are denoted by brown, blue and green colors.

Origin of 1-st order phase transition in HQCD. 1/3

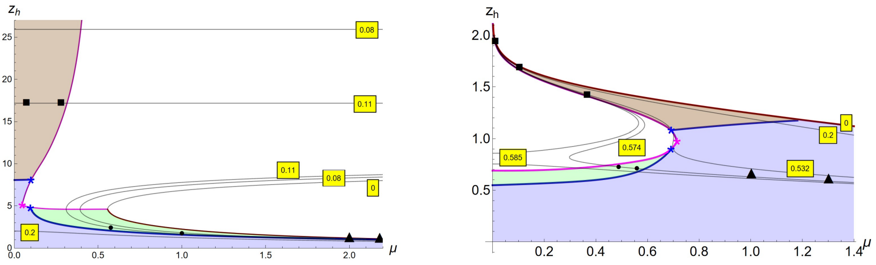


LQ



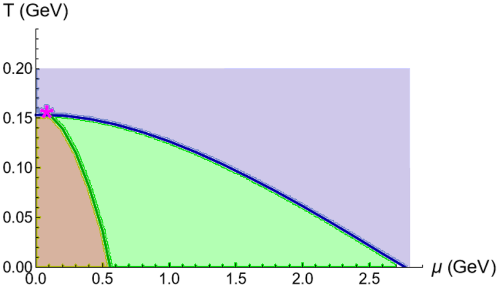
HQ

3D plot $T = T(\mu, z_h)$. The brown part of the surface corresponds to the hadronic phases, the blue one corresponds to the quark-gluon plasma and the green one to the quarkyonic phase. 3D plot $T = T(\mu, z_h)$

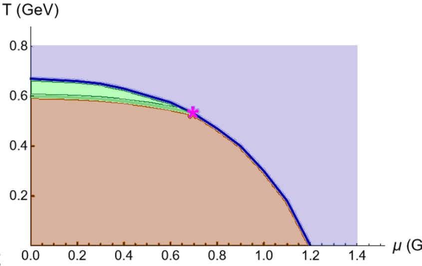


2D plots in (μ, z_h)-plane for light quarks (A) and heavy quarks (B). Hadronic, quarkyonic and QGP phases are denoted by brown, green and blue, respectively. Solid gray lines show the temperature indicated in rectangles. The intersection of the confinement/deconfinement and 1-st order phase transition lines is denoted by the blue stars. The magenta star indicates CEP

Origin of 1-st order phase transition in HQCD. 3/3



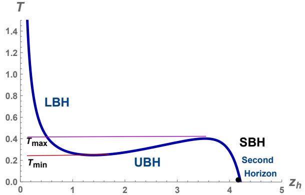
LQ

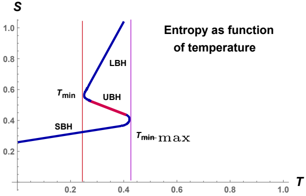


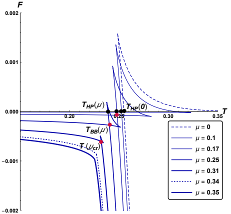
Origin of 1-st order phase transition in HQCD

- $g(z)$ blackenning function. The form of $g(z)$ depends on $\mathcal{A}(z)$.
- Due **non-monotonic** dependence of $T = T(z_h) = g'(z)/4\pi\Big|_{z=z_h}$ on z_h, the entropy $s = s(T)$ is **not monotonic**
- As a consequence the free energy $F = \int s dT$ undergoes the phase transition

1-st order phase transition describes transition
from small black holes $\rightarrow$ large black holes







The swallow-tailed shape

- Physical quantities that probe backgrounds are smooth relative to z_h
  $\Rightarrow$ their dependence on T should be taken from stable region
- Non-monotonic dependence of $T = T(z_h)$ gives the 1-st PT for corresponding characteristic of QCD

Conclusion

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- **Jumps of physical quantities, such as jet quenching, energy lost, etc.** on the 1-st order phase transition and dependence of jumps on B, anisotropy

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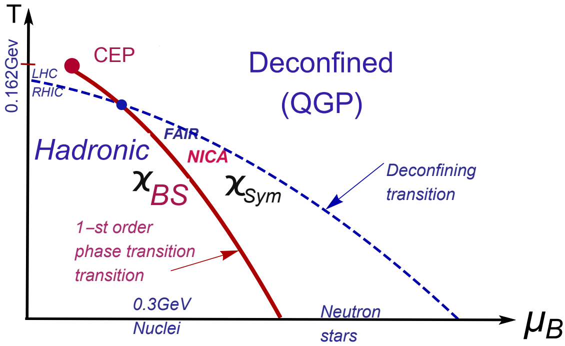
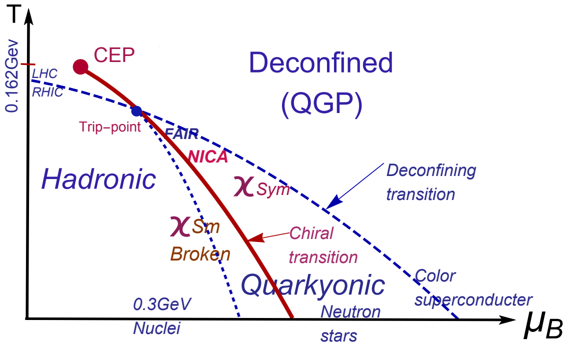
Backup. The expected more detailed QCD phase diagram



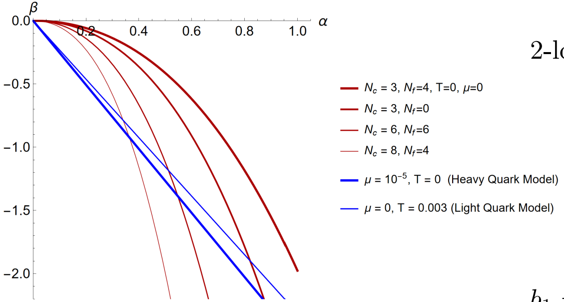
- Parameter of the chiral symmetry breaking $<\bar{\psi}\psi>$
  - $<\bar{\psi}\psi>=0 \Longleftrightarrow \chi$-symmetry
  - $<\bar{\psi}\psi>\neq 0 \Longleftrightarrow$ broken χ-symmetry

Backup. The expected more detailed QCD phase diagram



- Quarkyonic phase: baryon free ⇒ dense baryons *McLerran, Pisarski 0706.2191*
  - Baryon density jumps

Backup. β function



Beta-function $\beta(\alpha)$ for QCD at 2-loop level for $T = 0$, $\mu = 0$ at different N_c and N_f in red lines, and holographic β-function for light quarks at $\mu = 0$, $T = 0.003$ (light blue) and heavy quarks $\mu = 10^{-5}$, $T = 0$ (dark blue); $[\mu] = [T] =$ GeV.

2-loops QCD β-function:

$$\beta(\alpha) = -b_0\,\alpha^2 - b_1\,\alpha^3,$$

$$b_0 = \frac{1}{2\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

$$b_1 = \frac{1}{8\pi^2}\left(\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_f}\right)\right)$$

N_c # of colors, N_f #of flavors.

I.A, A. Hajilou, P. Slepov and M. Usova, arXiv:2407.14448